



# **Preliminary analysis of effect of random segment errors on coronagraph performance**

Mark T. Stahl, H. Philip Stahl,  
NASA Marshall Space Flight Ctr.

Stuart B. Shaklan,  
Jet Propulsion Laboratory  
California Institute of Technology



# Executive Summary

- Telescope manufacturers need a Wavefront Error (WFE) Stability Specification derived from science requirements.

## Wavefront Change per Time

- Develops methodology for deriving Specification.
- Develop modeling tool to explore effects of segmented aperture telescope wavefront stability on coronagraph.

## Caveats

- Monochromatic
- Simple model
- Band limited 4<sup>th</sup> order linear Sinc mask



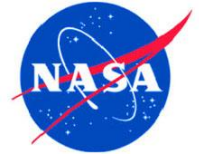
## Findings

- Reconfirms 10 picometers per 10 minutes Specification.
- Coronagraph Contrast Leakage is 10X more sensitivity to random segment piston WFE than to random tip/tilt error.
- Concludes that few segments (i.e. 0.5 to 1 ring) or very many segments ( $> 16$  rings) has less contrast leakage as a function of piston or tip/tilt than an aperture with 2 to 4 rings of segments.



# Aperture Dependencies

- Stability amplitude is independent of aperture diameter. It depends on required Contrast Stability as a function of IWA.
- Stability time depends on detected photon rate which depends on aperture, magnitude, throughput, spectral band, etc.
- For a fixed contrast at a fixed wavelength at a 40 mas angular separation, the wavefront stability requirement does have a ~4X larger amplitude for a 12-m telescope than for an 8-m telescope. And, it will have also have a shorter stability requirement for the same magnitude star.



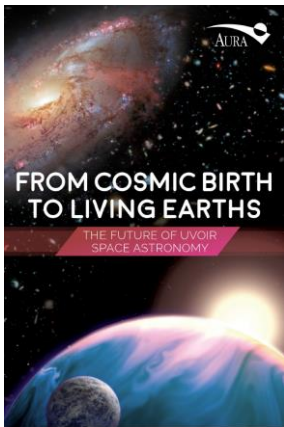
# Introduction



# Exoplanet Science

The search for extra-terrestrial life is probably the most compelling question in modern astronomy

The AURA report: From Cosmic Birth to Living Earths call for



A 12 meter class space telescope with sufficient stability and the appropriate instrumentation can find and characterize dozens of Earth-like planets and make transformational advances in astrophysics.

The key technical challenge is “sufficient stability” for the appropriate instrumentation.



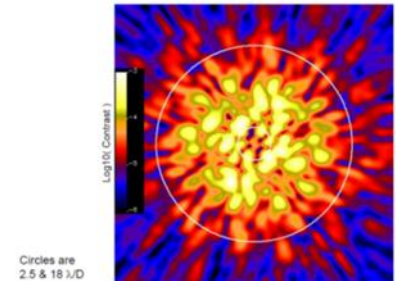
# ‘The’ System Challenge: Dark Hole

Imaging an ‘exo-Earth’ requires blocking  $10^{10}$  of host star’s light.

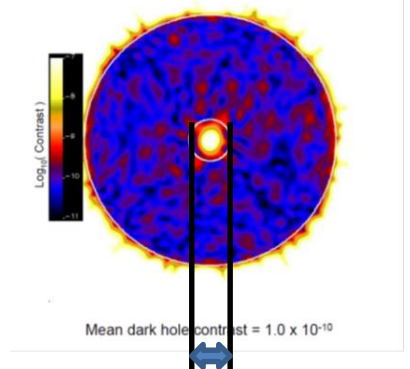
Internal coronagraph (with deformable mirrors) can create a ‘dark hole’ with  $< 10^{-10}$  contrast.

Once dark hole is established, the corrected wavefront phase must be kept stable to within a few picometers rms between science exposures to maintain the instantaneous (not averaged over integration time) speckle intensity to within  $10^{-11}$  contrast.

Contrast before wavefront correction  
 $\lambda = 500 - 600$  nm



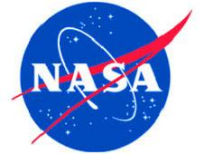
Contrast after correction  
 $\lambda = 500 - 600$  nm



Inner Working Angle  
(John Krist, JPL)

Krist, Trauger, Unwin and Traub, “End-to-end coronagraphic modeling including a low-order wavefront sensor”, SPIE Vol. 8422, 844253, 2012; doi: 10.1117/12.927143

Shaklan, Green and Palacios, “TPFC Optical Surface Requirements”, SPIE 626511-12, 2006.



# Wavefront Stability

Independent of Architecture (Monolithic or Segmented), any drift in WFE may result in speckles which can produce a false exoplanet measurement or mask a true signal.

Important WFE stability sources include relative misalignment motion between optical components or shape changes of individual optical components or their support structures.

There are 2 primary source of Temporal Wavefront Error:

- Thermal Environment

- Mechanical Environment





# Wavefront Stability - Thermal

Changes in orientation relative to the Sun changes the system thermal load. These changes can increase (or decrease) the average temperature and introduce thermal gradients.

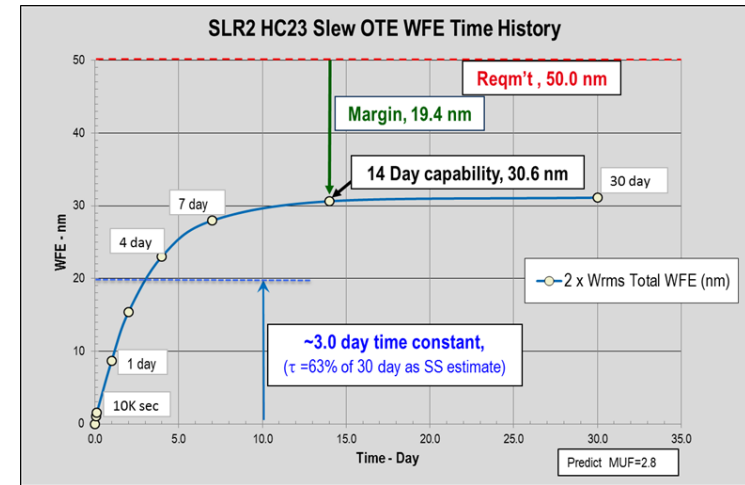
In response to the 'steady-state' temperature change, variations in the Coefficient of Thermal Expansion (CTE) distribution cause static wavefront errors.

Stability errors depend on the temporal response of the mirror system to the thermal change, i.e. depends on mirror material.

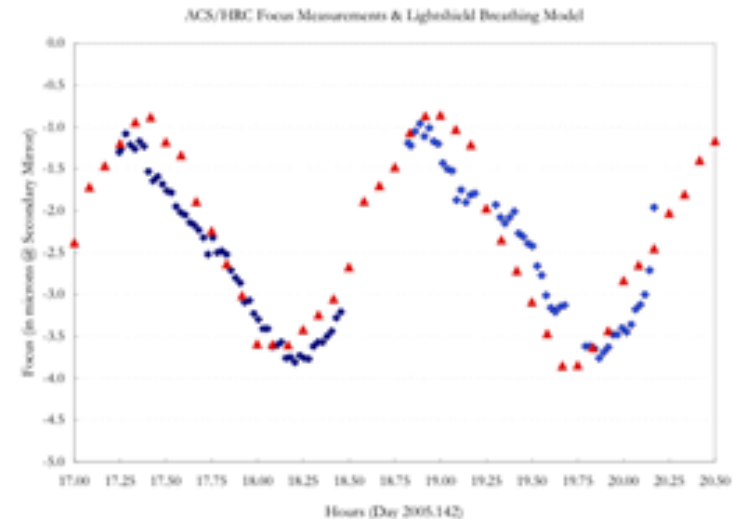


# Wavefront Stability - Thermal

For example, (while not designed for a UVOIR Exoplanet Science Mission) JWST experiences a worst-case thermal slew of 0.22K which results in a 31 nm rms WFE response. And it takes 14 days to ‘passively’ achieve  $< 10$  pm per 10 min



HST (which is a cold-biased telescope heated to an ambient temperature) WFE changes by 10–25 nm every 90 min (1–3 nm per 10 min) as it goes in and out of Earth’s shadow.





# Wavefront Stability - Mechanical

For example, (while not designed for a UVOIR Exoplanet Mission) JWST has a predicted WFE response of  $< 13$  nm rms for temporal frequencies up to 70 Hz.

JWST has several mechanical modes:

- PMA structure has  $\sim 40$  nm rms 'wing-flap' mode at  $\sim 20$  Hz
- Individual PMSAs have  $\sim 20$  nm rms 'rocking' mode at  $\sim 40$  Hz

To meet a 10 pm stability specification requires these amplitudes to be reduced by 1000X.

JWST engineers (private conversation) estimate this could be done by combination of passive and active methods:

- Ambient telescope will have 10X more damping.
- Structure can be made stiffer
- 140 dB of active vibration isolation (JWST has  $\sim 90$  dB of isolation)



# System Alignment

Misalignments produce low-order errors

- Lateral De-center between PM and SM produce Siedel Coma
- Tilt produces Siedel Astigmatism
- De-space produces Siedel Focus and Siedel Spherical

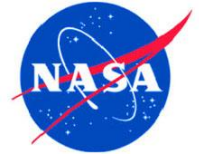
Siedel aberrations because system does not ‘refocus’ or adjust ‘tilt’ in real time to compensate for these errors.

Allocation	Drift
Rigid Body Pointing (FSM Compensated)	1 mas/axis
Drift of Image on Mask	5 uas/axis
SM Motion relative to PM	
x, y rotation	0.5 nrad
x, y translation	0.5 nm
z translation (focus)	0.1 nm
Instrument motion relative to PM	
rotation	5 nr/axis
translation	5 nm/axis
PM aberrations	
Focus	1 pm
Coma	1 pm
Spherical	1 pm

$$\Delta WFE < 1 \text{ pm}$$

Deformable Mirrors typically correct for these errors.

BUT, if these alignment errors are dynamic



## What is the right $\Delta WFE$ Stability Requirement?

Depends on Amplitude Sensitivity and  
Controllability



# 10 picometers per 10 minutes Wavefront Stability

In AMTD-1 2013 paper we:

- Proposed  $\Delta WFE < 10$  pm per 10 minute Specification.
- And, considered Wavefront Stability issues of a Segmented Mirror

In AMTD-2 2014 paper we:

- Refined 10 pm per 10 minute Wavefront Error Stability Specification.
- Discuss the scaling of Aperture Size and Stiffness effect on Stability.

Stahl, H. Philip, Marc Postman and W. Scott Smith, “Engineering specifications for large aperture UVO space telescopes derived from science requirements”, Proc. SPIE 8860, 2013, DOI: 10.1117/12.2024480

Stahl, H. Philip Stahl; Marc Postman; Gary Mosier; W. Scott Smith; Carl Blaurock; Kong Ha; Christopher C. Stark, “AMTD: update of engineering specifications derived from science requirements for future UVOIR space telescopes”, Proc. SPIE. 9143, Space Telescopes and Instrumentation 2014: Optical, Infrared, and Millimeter Wave, 91431T. (August 02, 2014) doi: 10.1117/12.2054766



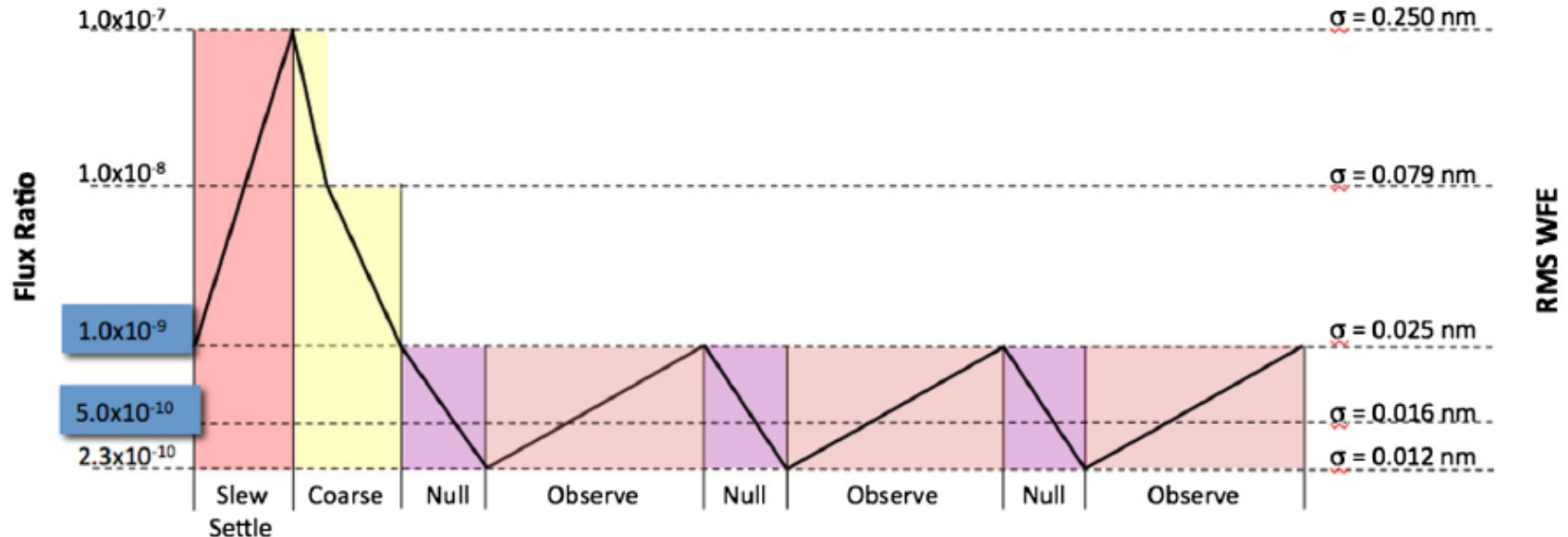
# Controllability



# Primary Mirror Surface Figure Error Stability

Per Lyon and Clampin:

- If the telescope system cannot be designed near zero stability, then the WFE must be actively controlled.
- Assuming that DMs perfectly ‘correct’ WFE error once every ‘control period’, then the Telescope must have a WFE change less than the required ‘few’ picometers between corrections.



Lyon and Clampin, “Space telescope sensitivity and controls for exoplanet imaging”, Optical Engineering, Vol 51, 2012; 011002-2





# Co-Phasing Stability vs Segmentation

Per Guyon:

- Co-Phasing required to meet given contrast level depends on number of segments; is independent of telescope diameter.
- Time required to control co-phasing depends on telescope diameter; is independent of number of segments.
  - To measure a segment's co-phase error takes longer if the segment is smaller because there are fewer photons.
  - But, allowable co-phase error is larger for more segments.

TABLE 1: Segment cophasing requirements for space-based telescopes  
(wavefront sensing done at  $\lambda=550\text{nm}$  with an effective spectral bandwidth  $\delta\lambda=100\text{ nm}$ )

Telescope diameter (D) & $\lambda$	Number of Segments (N)	Contrast	Target	Cophasing requirement	Stability timescale
4 m, 0.55 $\mu\text{m}$	10	1e-10	$m_V=8$	2.8 pm	22 mn
8 m, 0.55 $\mu\text{m}$	10	1e-10	$m_V=8$	2.8 pm	5.4 mn
8 m, 0.55 $\mu\text{m}$	100	1e-10	$m_V=8$	8.7 pm	5.4 mn

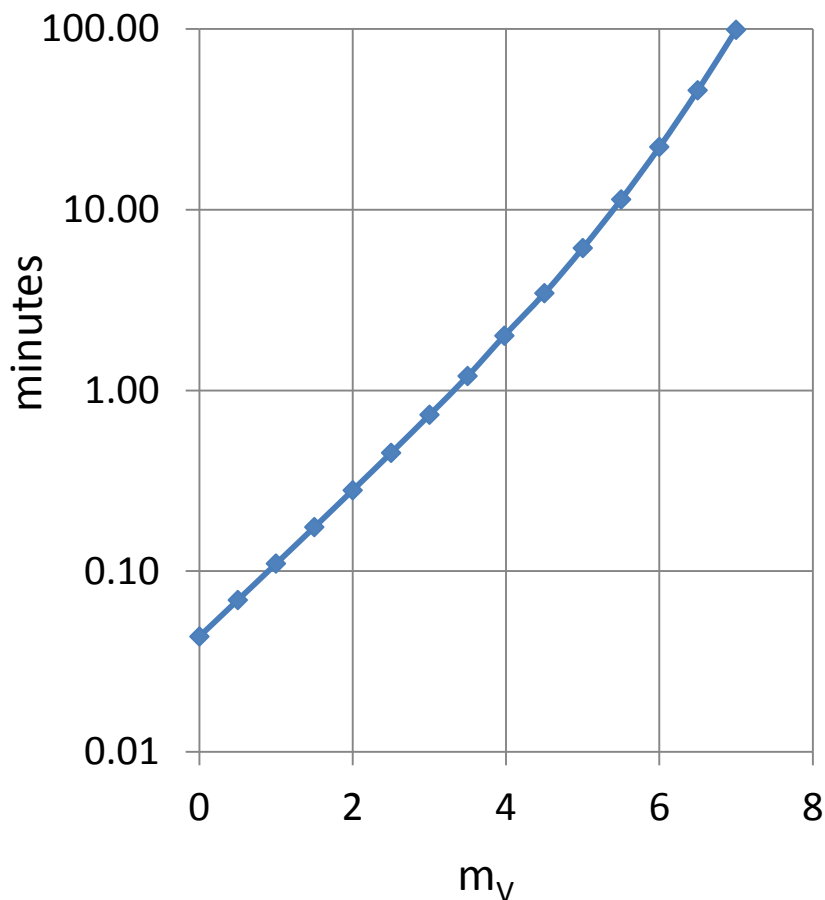


# Controllability Period

- Krist (Private Communication, 2013): wavefront changes of the first 11 Zernikes can be measured with accuracy of 5 – 8 pm rms in 60 – 120 sec on a 5<sup>th</sup> magnitude star in a 4 m telescope over a 500 – 600 nm pass band (reflection off the occulter). This accuracy scales proportional to square root of exposure time or telescope area.
- Lyon (Private Communication, 2013): 8 pm control takes ~64 sec for a Vega 0<sup>th</sup> mag star and 500 – 600 nm pass band [ $10^8$  photons/m<sup>2</sup>-sec-nm produce  $4.7 \times 10^5$  electrons/DOF and sensing error  $\sim 0.00073$  radians = 64 pm at  $\lambda = 550$  nm]
- Guyon (Private Communication, 2012): measuring a single sine wave to 0.8 pm amplitude on a Magnitude V=5 star with an 8-m diameter telescope and a 100 nm effective bandwidth takes 20 seconds. [Measurement needs  $10^{11}$  photons and V=5 star has  $10^6$  photons/m<sup>2</sup>-sec-nm.] BUT, Controllability needs 3 to 10 Measurements, thus stability period requirement is 10X measurement period.



# Integration Time for a 10 m telescope



## Simulation Parameters

$\Delta\text{mag} = 25$  (to control background level)

Spectral Resolution = 10

SNR = 3 per channel

Throughput 42%

QE 80%

No detector noise

Instrument contrast =  $1e-10$

Zodi + exozodi =  $3 \times 23$  mag/sq. arcsec

Wavelength 760 nm

Sharpness 0.08



# Primary Mirror SFE Stability Specification

Telescope and PM must be stable  $< 10$  pm for periods longer than the control loop period.

The exact length of the control period length depends on

- Aperture Diameter of Telescope

- 'Brightness' of Star used to sense WFE

- Spectral Bandwidth of Sensing

- Spatial Frequency Degrees of Freedom being Sensed

- Wavefront Control 'Overhead' and 'Efficacy'

In general, it seems like a 'good' consensus requirement is:

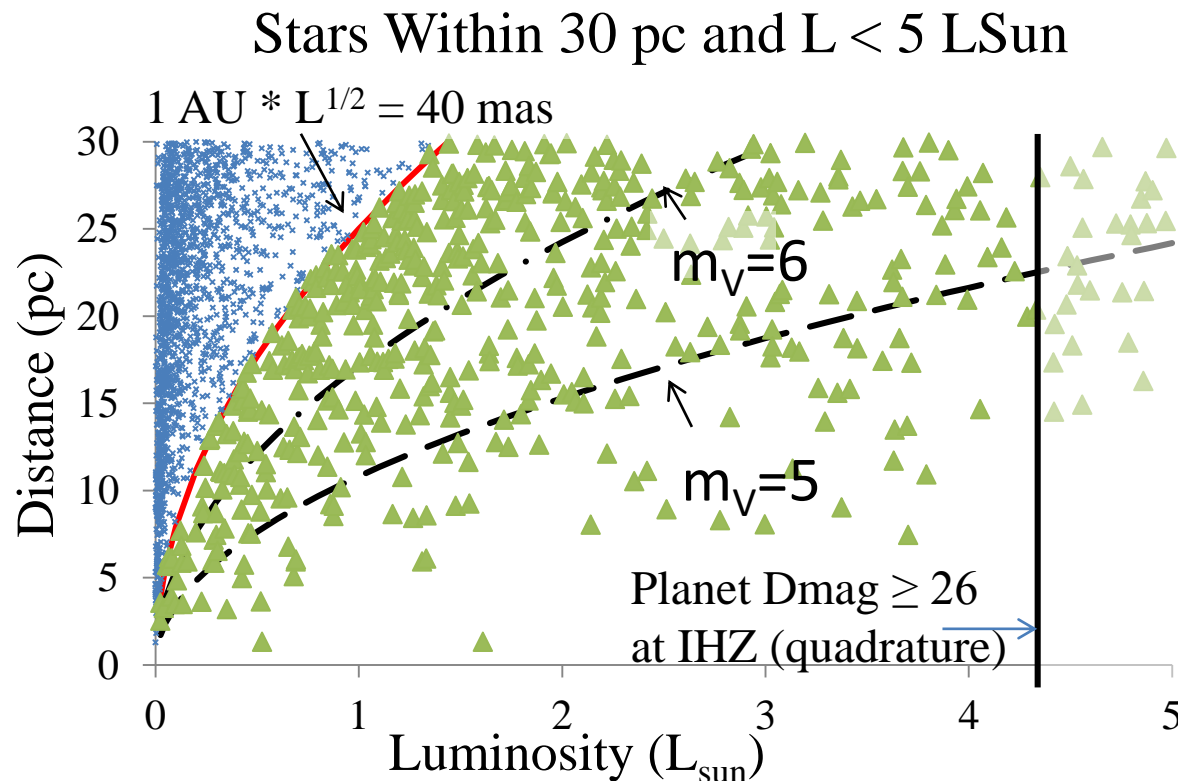
$< 10$  picometers per 10 minutes



# Consequence of Controllability

There may be a practical limit to the telescope aperture size based on inner working angle needed to search dimmest star for which the control loop can be closed.

Could make Aperture larger to reduce time, but this is less stable.





# Problem of Aperture

Per the AURA “Cosmic Birth to Living Earth” report, the science community desires a 12-m class collecting aperture.

To achieve a 12-meter class aperture requires segmentation.

Segmented apertures have many challenges:

- Prescription Matching
- Segmentation Pattern results in secondary peaks
- Segmentation Gaps redistribute energy
- Rolled Edges redistribute energy
- Segment Co-Phasing Absolute Accuracy
- Segment Co-Phasing Stability

To do exoplanet science requires that the segmented telescope must be co-phased.

To meet the  $10^{-11}$  contrast stability requirement requires that the telescope co-phasing is stable.



# Segmented Aperture Point Spread Function (PSF)

For perfectly phased telescope with no gaps & optically perfect segments, zeros of  $\text{PSF}_{\text{seg}}$  coincide with peaks of Grid function resulting in  $\text{PSF}_{\text{tel}}$  with a central peak size  $\sim \lambda/D_{\text{tel}}$

In a real telescope: gaps, tip/tilt errors, piston errors, rolled edges & figure errors move energy from the central core to higher-order peaks and into the speckle pattern.

1566 J. Opt. Soc. Am. A/Vol. 20, No. 8/August 2003

Yaitskova *et al.*

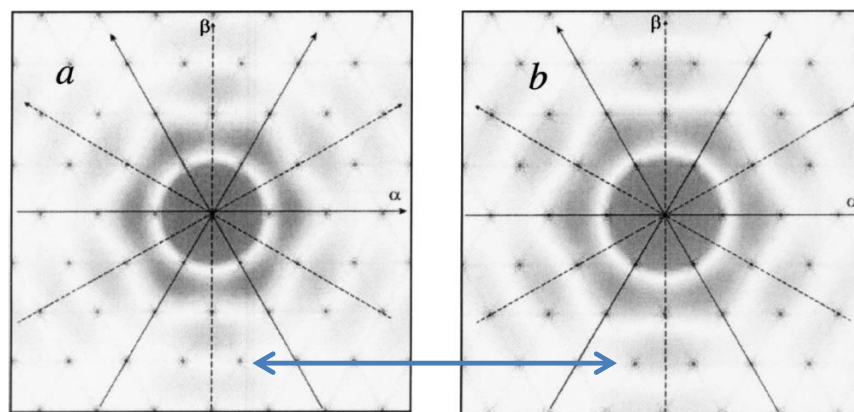


Fig. 2. a, Grid factor (regular spots) and the segment  $\text{PSF}_s$  for a perfect telescope without gaps. Except for the central peak, all peaks of the grid factor fall into zeros of the segment  $\text{PSF}_s$ . Solid and dashed arrows illustrate the same double  $\pi/3$  symmetry as observed in the pupil plane (Fig. 1). b, The same, but with gaps between segments (relative gap size  $\omega=0.1$ ). Higher-order peaks are no longer coincident with  $\text{PSF}_s$  zeros. The same effect is seen for tip-tilt errors and segment-edge misfigure.



# Tip/Tilt Errors

A segmented aperture with tip/tilt errors is like a blazed grating removes energy from central core to higher-order peaks.

If the error is ‘static’ then a segmented tip/tilt deformable mirror should be able to ‘correct’ the error and any residual error should be ‘fixed-pattern’ and thus removable from the image.

But, if error is ‘dynamic’, then higher-order peaks will ‘wink’.

Yaitskova *et al.* J. Opt. Soc. Am. A/Vol. 20, No. 8/August 2003

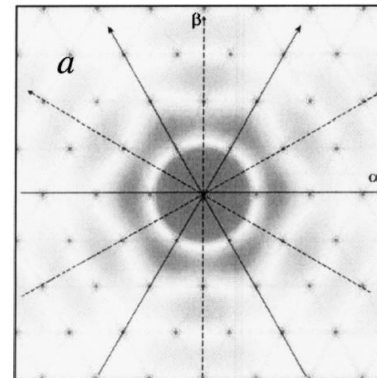
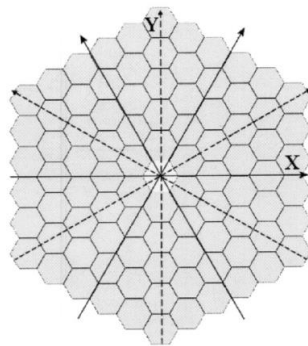


Fig. 1. Segmented mirror with segmentation order  $M = 5$  consisting of  $N = 90$  segments. Solid and dashed arrows illustrate the double  $\pi/3$  symmetry of the system.





# Co-Phasing Errors

Co-Phasing errors introduce speckles whose size is inversely proportional to the segment size.

If the error is ‘static’ then a segmented piston deformable mirror should be able to ‘correct’ the error and any residual error should be ‘fixed-pattern’ and thus removable from the image.

But, if error is ‘dynamic’, then speckles will move.

Yaitskova *et al.* J. Opt. Soc. Am. A/Vol. 20, No. 8/August 2003

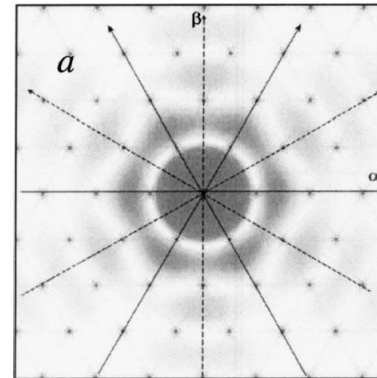
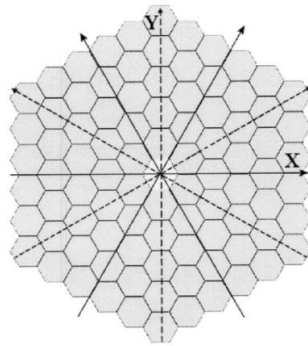


Fig. 1. Segmented mirror with segmentation order  $M = 5$  consisting of  $N = 90$  segments. Solid and dashed arrows illustrate the double  $\pi/3$  symmetry of the system.



# Segmentation vs. Dark Hole

In our 2013 paper, we asked the question:

Is fewer large segments better or is many small better?

The context of the question related to the idea of engineering the aperture to place the diffraction orders inside the dark hole inner working angle or outside of the dark hole outer working angle.

At Mirror Technology Days 2014, Stuart Shaklan reported on a preliminary answer to this question.

Based on Contrast Leakage for piston or tip/tilt error as a function of number of segments in a square aperture, it is better to have less than 4 segments per diameter or more than 32 segments per diameter.

This paper seeks to continue this effort.

Stahl, H. Philip, Marc Postman and W. Scott Smith, “Engineering specifications for large aperture UVO space telescopes derived from science requirements”, Proc. SPIE 8860, 2013, DOI: 10.1117/12.2024480

Shaklan, “Segmented Telescope Stability Error Budget for Exo-Earth Direct Imaging”, Mirror Tech Days, 2014. 26



# Contrast vs. Number of Segments

## Square telescope

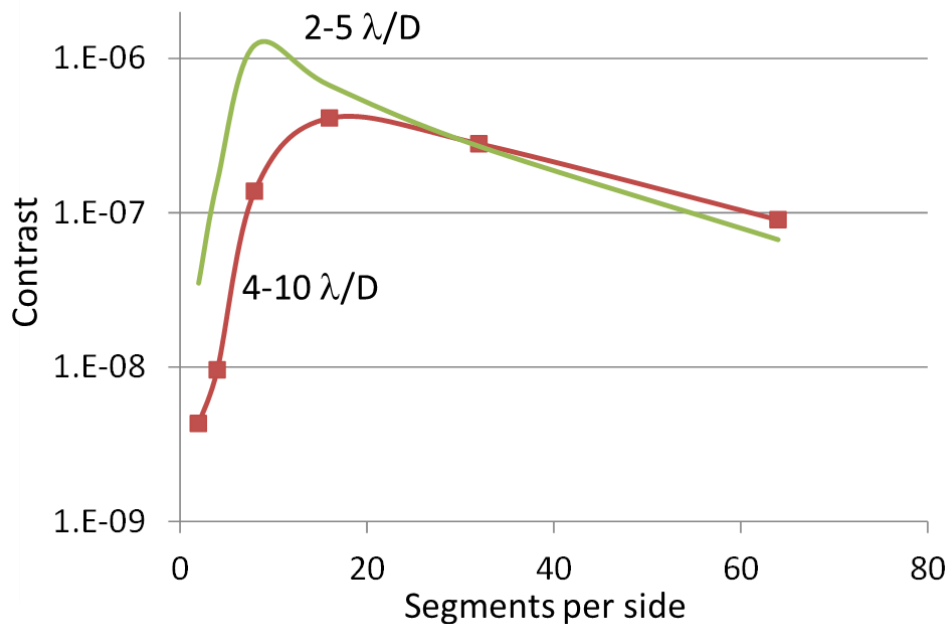
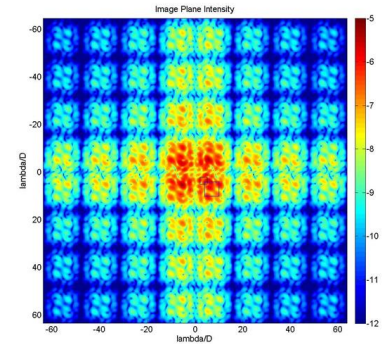
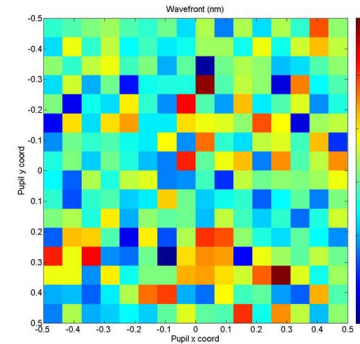
2x2, 4x4, 8x8...64x64 segments

1 nm piston rms random per segment

## Coronagraph

$\lambda = 600$  nm

2-D 1-sinc<sup>2</sup> Mask with 1<sup>st</sup> Transmission mas at  $4 \lambda/D$





# Contrast vs. Number of Segments

## Square telescope

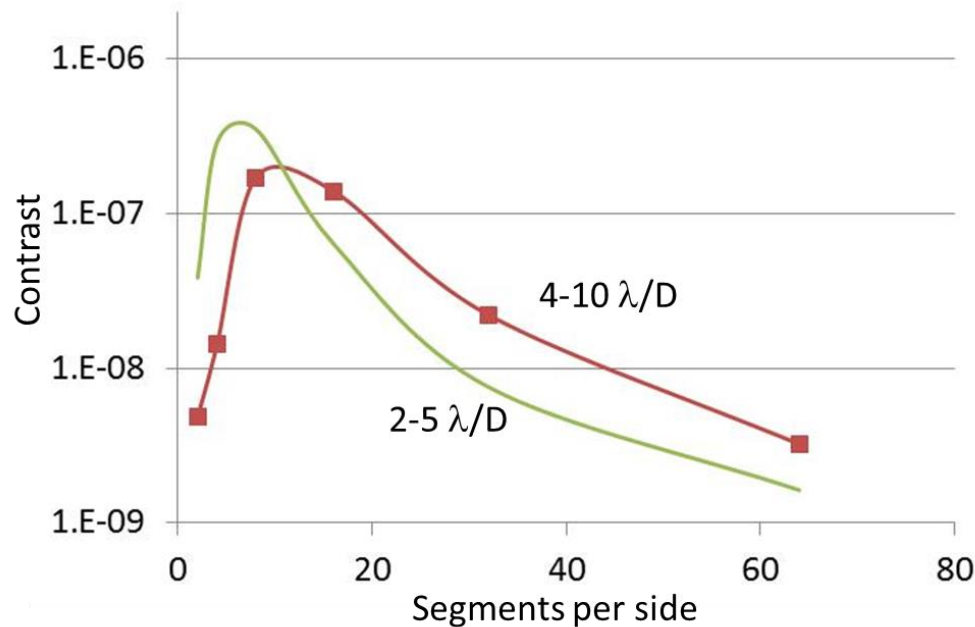
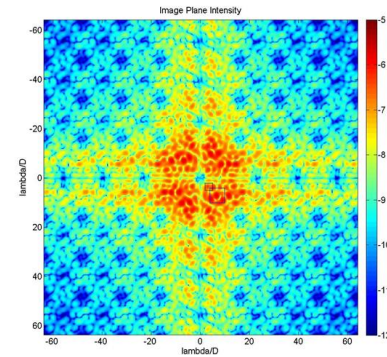
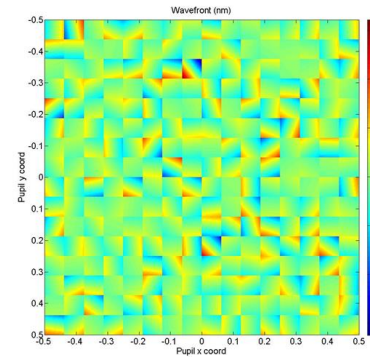
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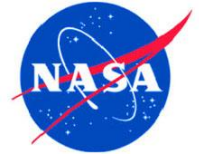
1 nm tip-tilt rms random per segment

## Coronagraph

$\lambda = 600$  nm

2-D 1-sinc<sup>2</sup> Mask with 1<sup>st</sup> Transmission mas at  $4 \lambda/D$





# Segmented Aperture Stability Requirement

This paper explores the stability requirements for a segmented aperture telescope for use with an internal coronagraph.

Stability sensitivity as a function of segmentation is reported.

Methodology is to create an integrated model of a segmented aperture telescope and a band-limited mask coronagraph

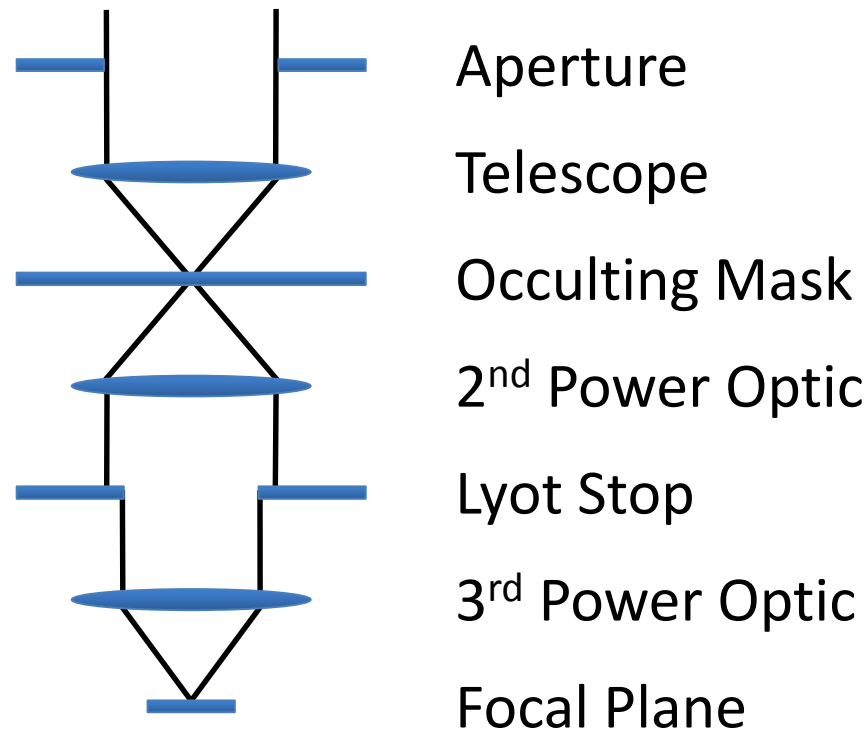


# Integrated Model

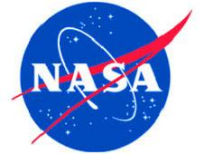


# Integrated Model

Using Matlab, we created an integrated model of a segmented aperture telescope and a single stage internal linear band-limited coronagraph:  $\{1 - \text{sinc}^2(x)\}$ .



Note: We are using  $\gamma = 6$  zero padding



# Integrated Model – Pupil Function

Pupil Function models the telescope

$$\text{Pupil}(x,y) = \text{Aper}(x,y) * \text{Phase}(x,y) = A(x,y)e^{-i\Phi(x,y)}$$

## Aperture Mask

- Can model Monolithic or Segmented Aperture
- Segments can be Hexagonal or Square
- Outer Aperture can be Hex Segment Boundary or Circle
- Square segmentation pattern from 1x1, 2x2, 4x4, .... 512x512
- Hex segmentation pattern is 0, 1, 2, ... to 6 Rings.
- Gaps for Square Segments Only (ignore for this study)
- Can also do Central Circular Obscuration and 'cross' spiders

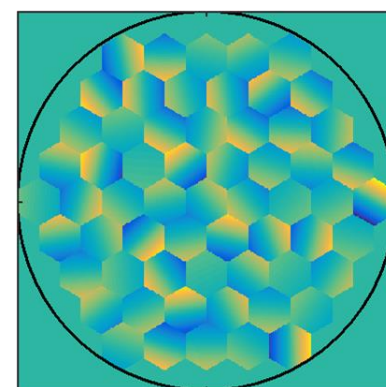
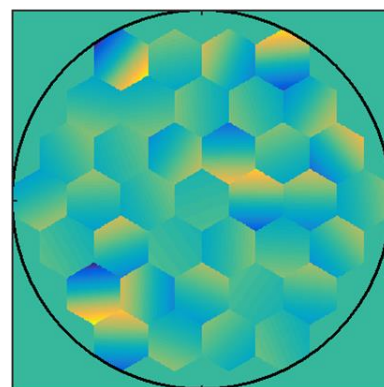
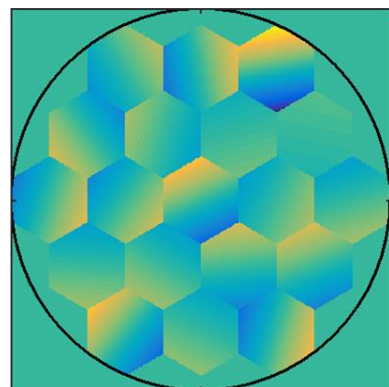
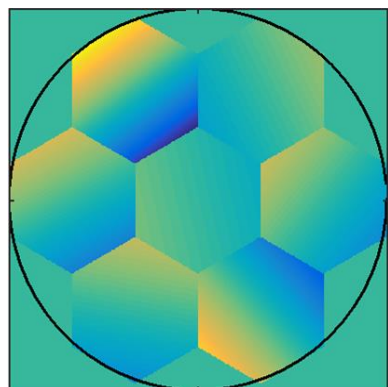
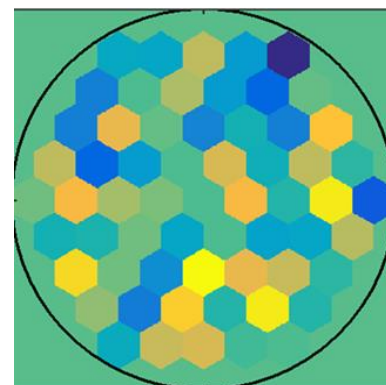
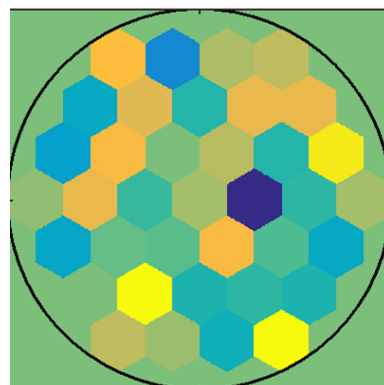
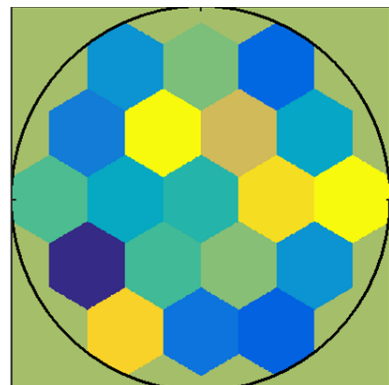
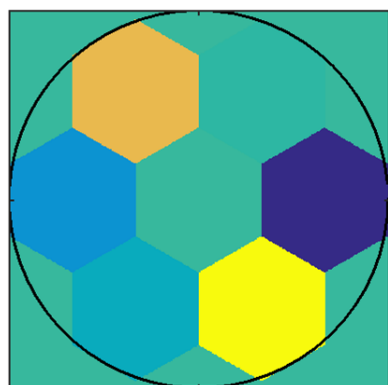
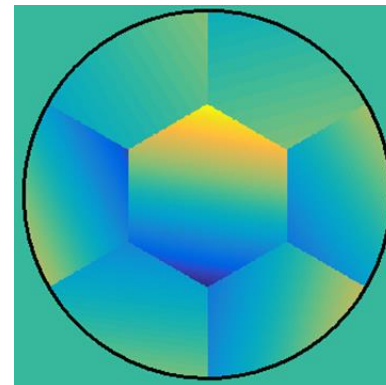
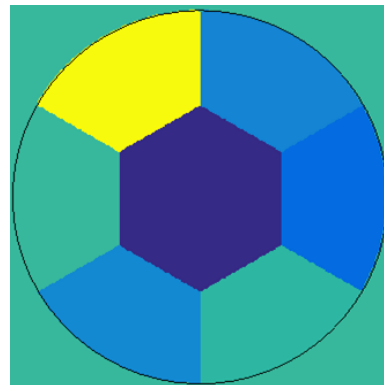
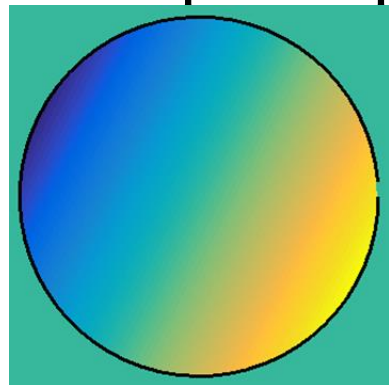
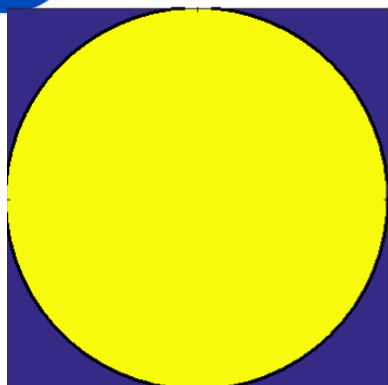
## Phase defines telescope Wavefront Error

- Random Segment Rigid Body: piston, tip/tilt (assume that a DM corrects any slow or systematic error)





# Input Pupil Functions



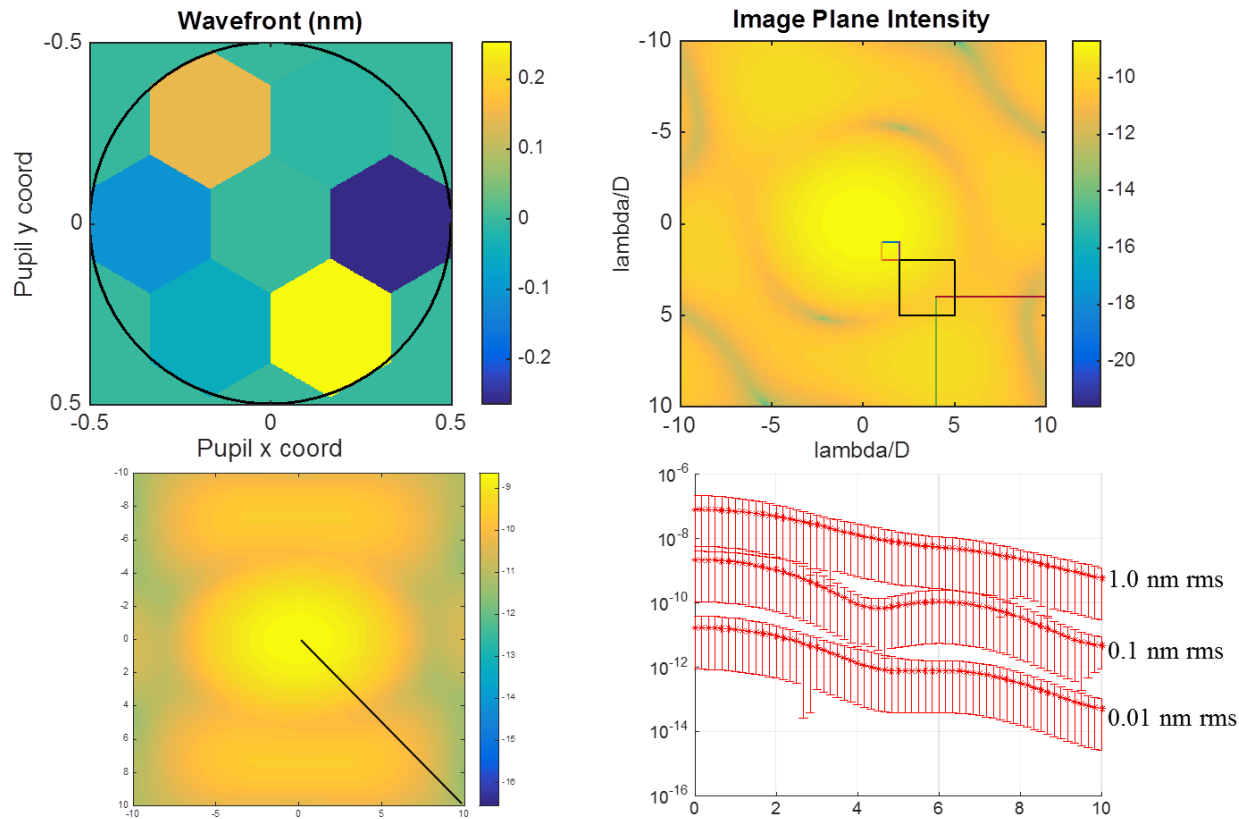


# Integrated Model – Output

Output is Contrast (single realization & N average)

- 2D Plot
- 1D Profile
- Average inside ROI from  $1-2 \lambda/D$ ,  $2-5 \lambda/D$  &  $4-10 \lambda/D$

$N_{\text{hex}} = 1 \ 1/0.1/0.01$  rms piston  $N = 16$ ,  $\gamma = 6$

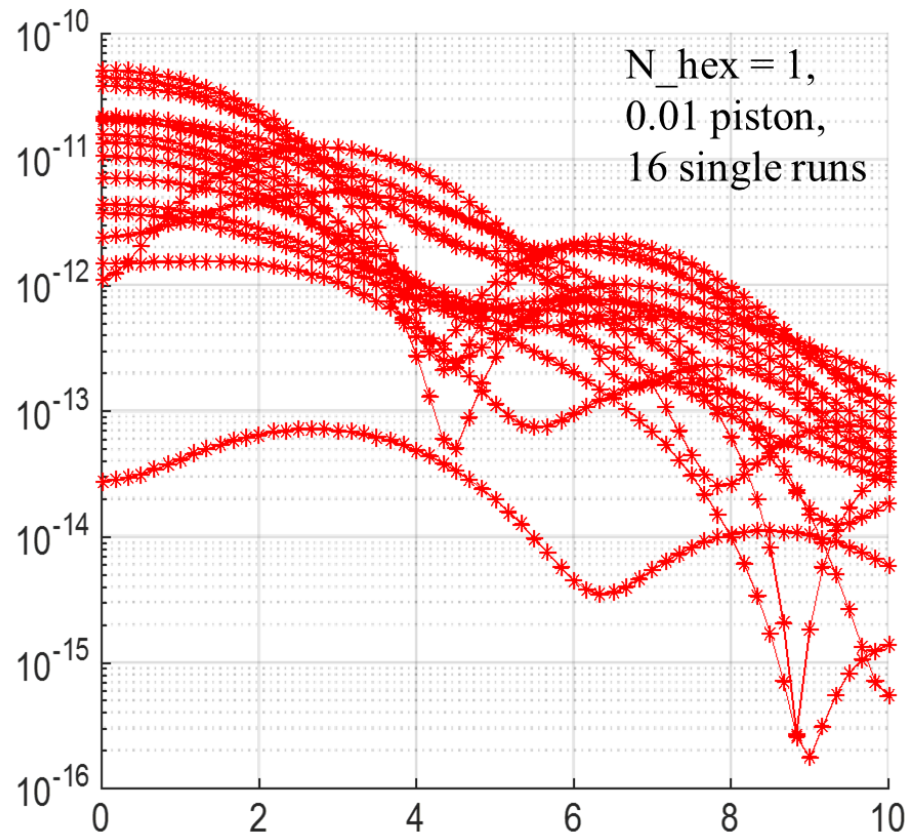


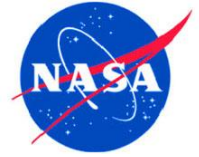


# Error Bars

Error Bars on the 1D profile show the range of contrast values for the N individual model realizations.

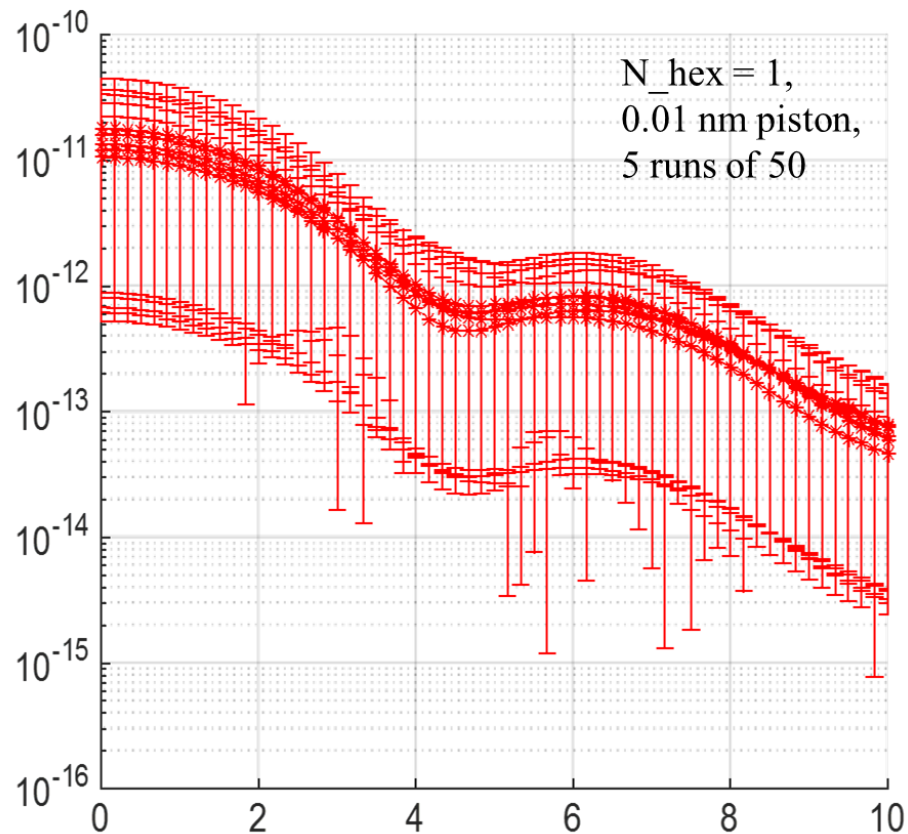
Below is 16 individual realizations for a 1 ring Hex aperture with 0.10 nm rms random piston error.





# Reproducibility

Even with averaging 50 individual realizations, there is still some variability in the result

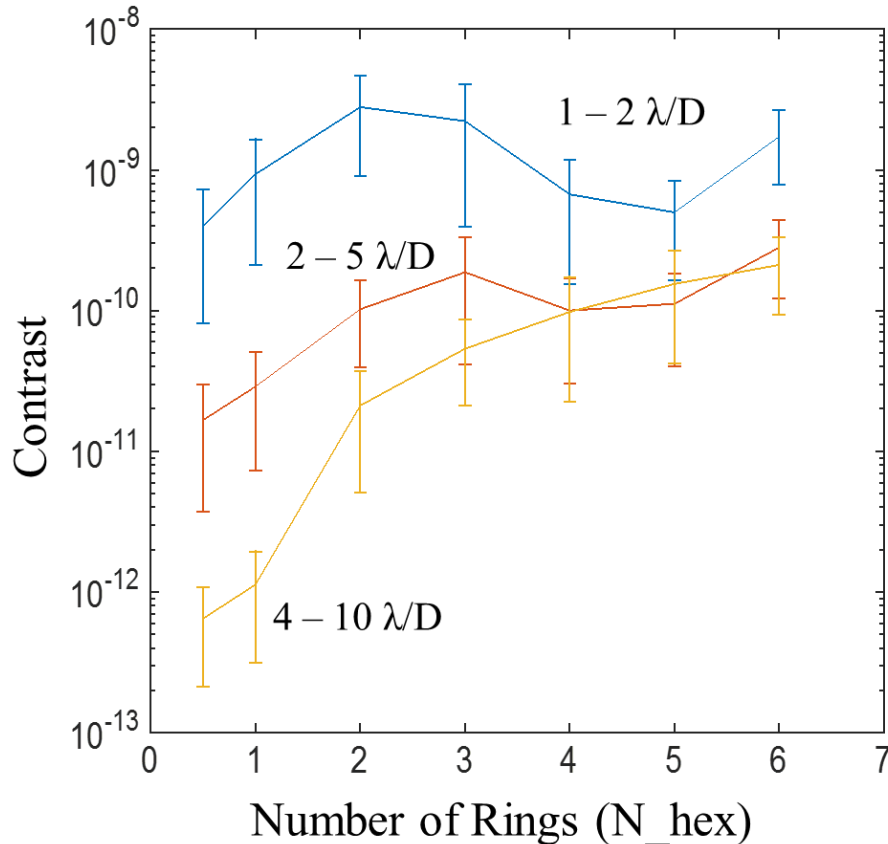




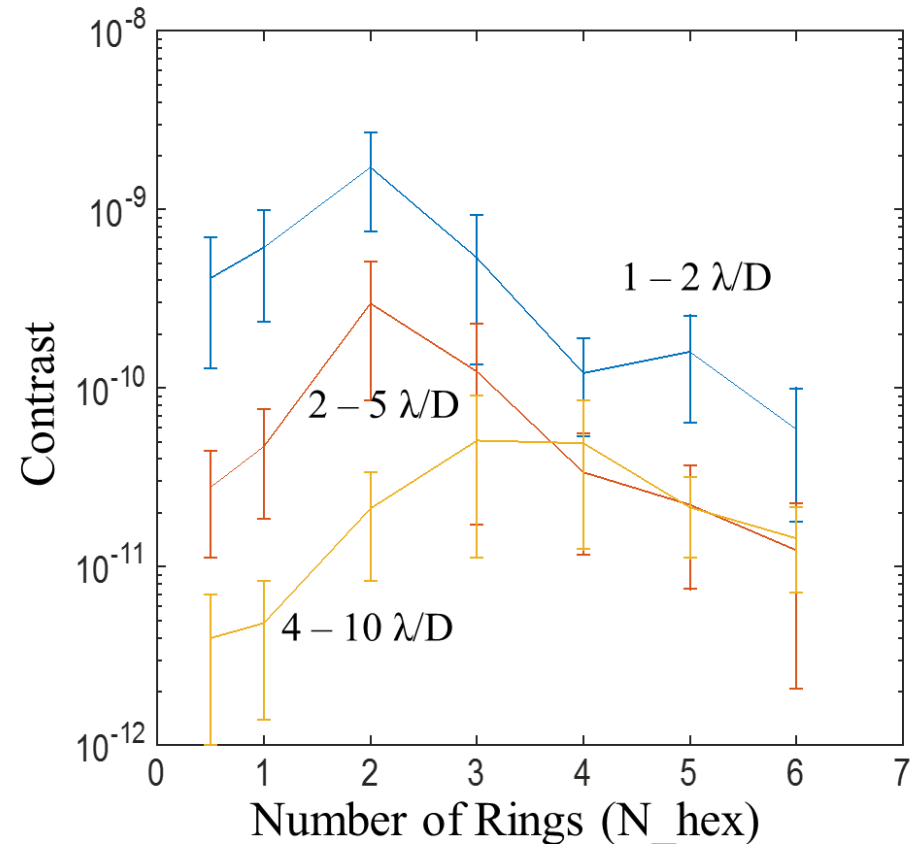
# WFE Sensitivity vs Number of Segments

- Contrast Leakage is 10X more sensitive to Piston than Tilt.
- Contrast Leakage is less for fewer segments

Contrast for 0.01 nm Piston vs.  $\lambda/D$



Contrast for 0.1 nm Tip/Tilt vs.  $\lambda/D$



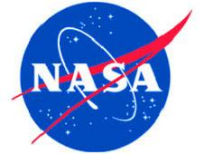


## Future Work



## Planned Future Work

- Random Seidel Power on each Segment for thermal drift
- Correlated Tip-Tilt Segment motion
- Add System Alignment Aberration:
  - Seidel Coma to simulate PM/SM alignment
  - Seidel Astigmatism to simulate PM Structure 'flapping'
- Add a Planet
- Add more Central Obscuration options
- Add more SM Spider options
- Other Occulting Masks: Gauss, Sine, etc.



# Conclusions





## Conclusions

Developed modeling tool to explore effects of a segmented aperture telescope on a band-limited mask coronagraph.

Coronagraph Contrast Leakage is 10X more sensitivity to random segment piston WFE than to random tip/tilt error.

Coronagraph Contrast Leakage is less for Fewer Segments.

A 'conservative' WFE Stability Requirement continues to be:

**10 picometers per 10 minutes**

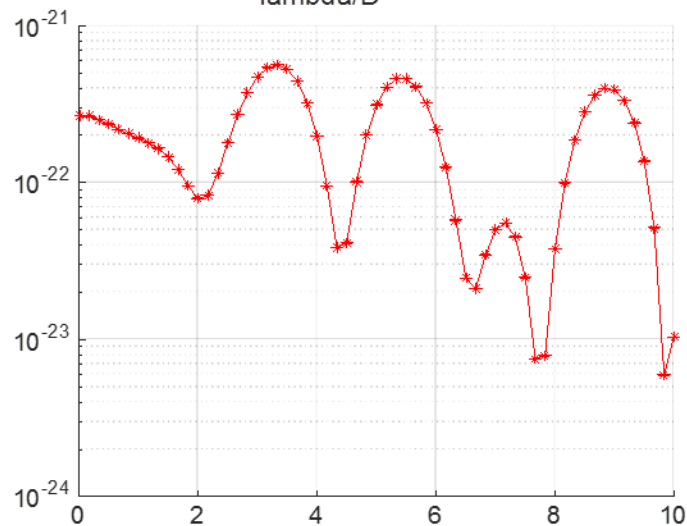
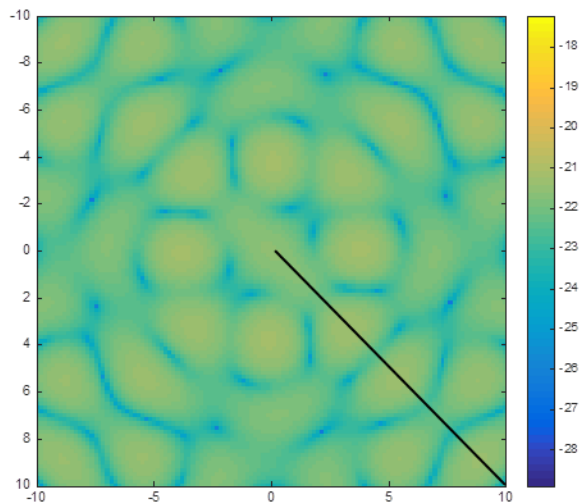
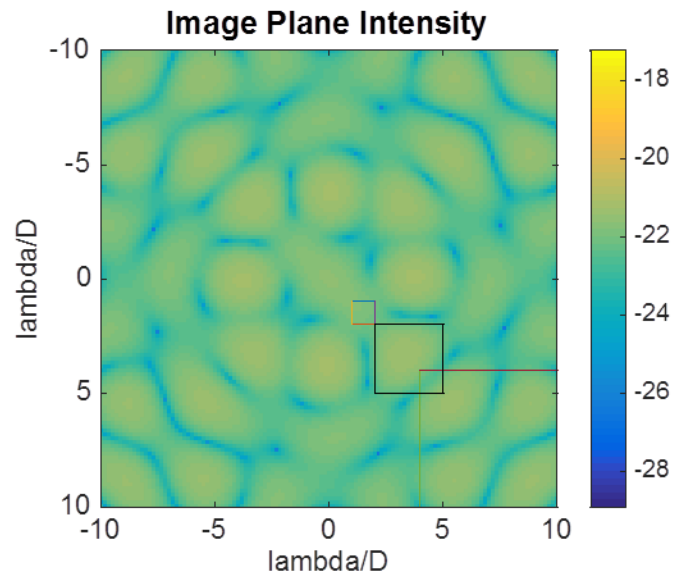
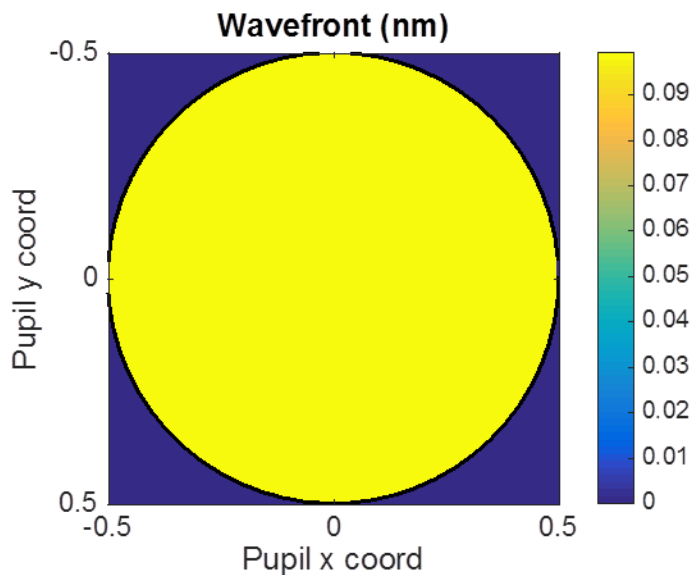


BACKUP



1x1 0.1 rms piston  $N = 16$ ,  $\gamma = 6$

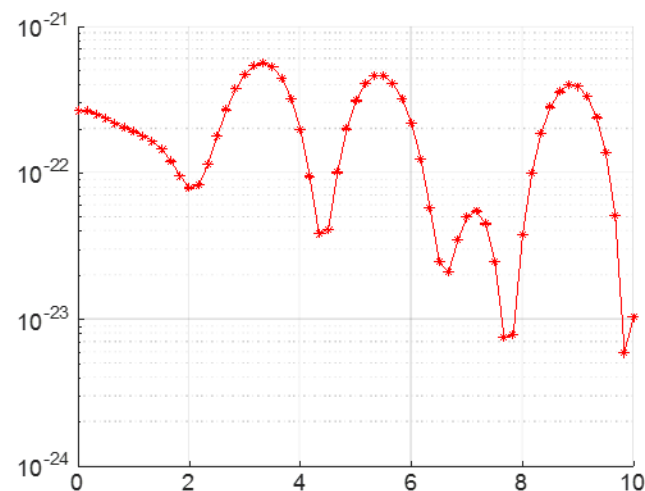
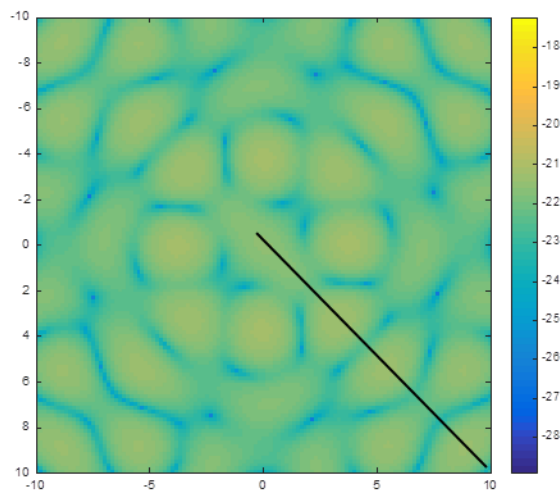
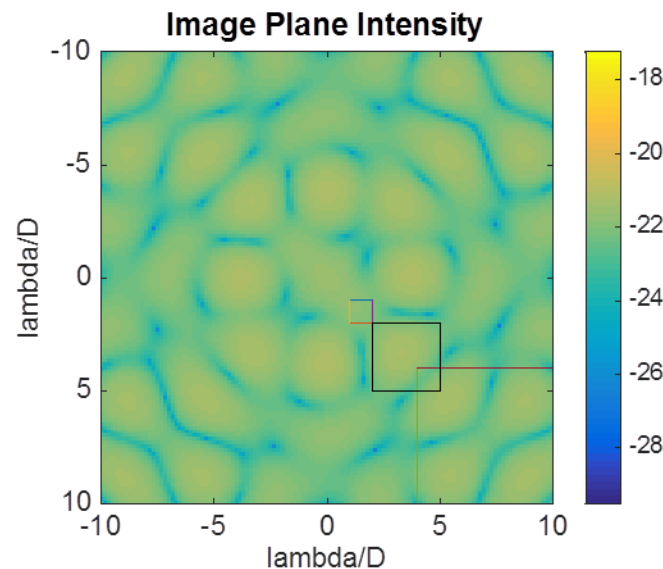
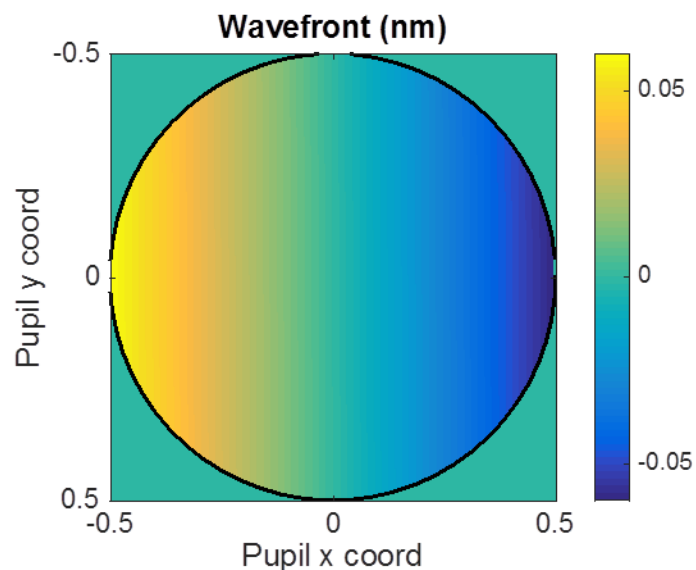
For monolith, piston yields essentially a 'perfect' system





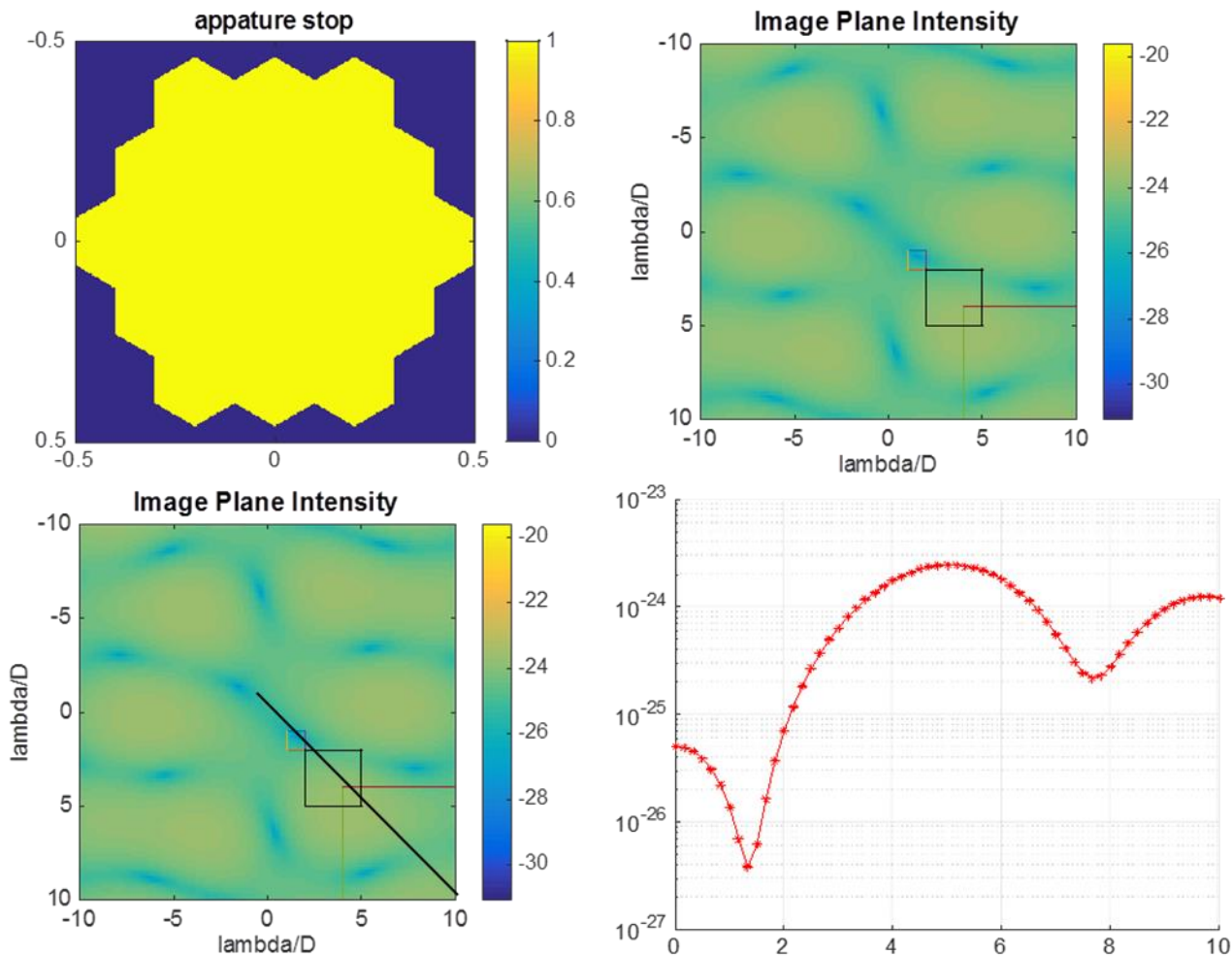
1x1 0.1 rms tip/tilt  $N = 16$ ,  $\gamma = 6$

For monolith, tip/tilt yields essentially a 'perfect' system





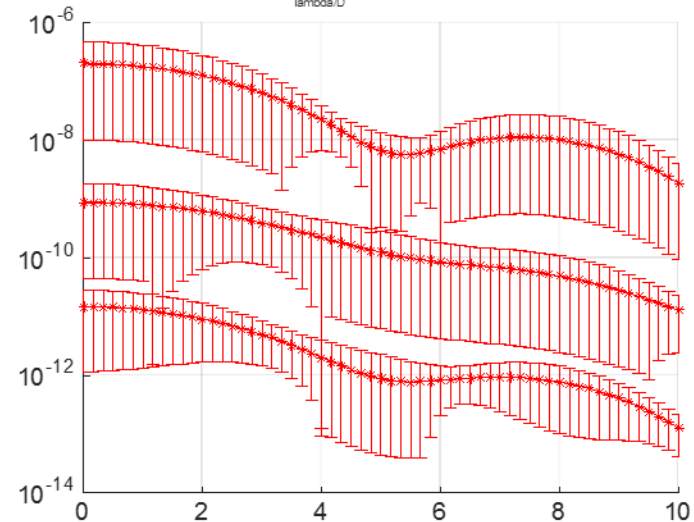
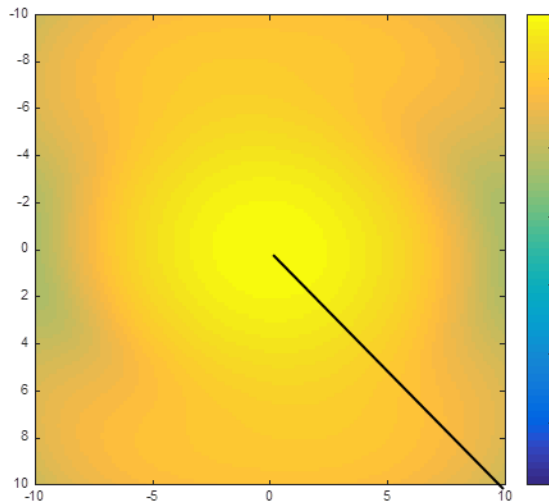
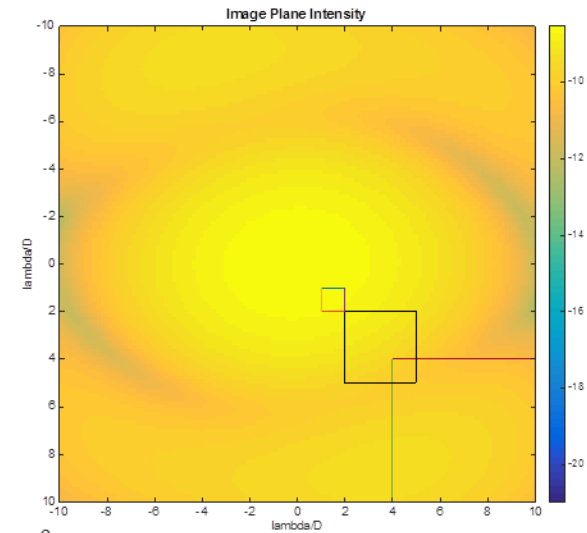
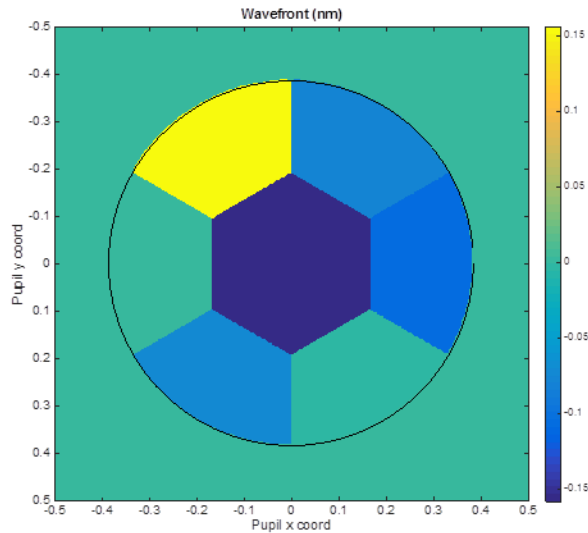
## 2 Ring Hex Aperture with no Wavefront Error





$N_{\text{hex}} = 1 \ 1/0.1/0.01$  rms piston  $N = 16, \gamma = 6$

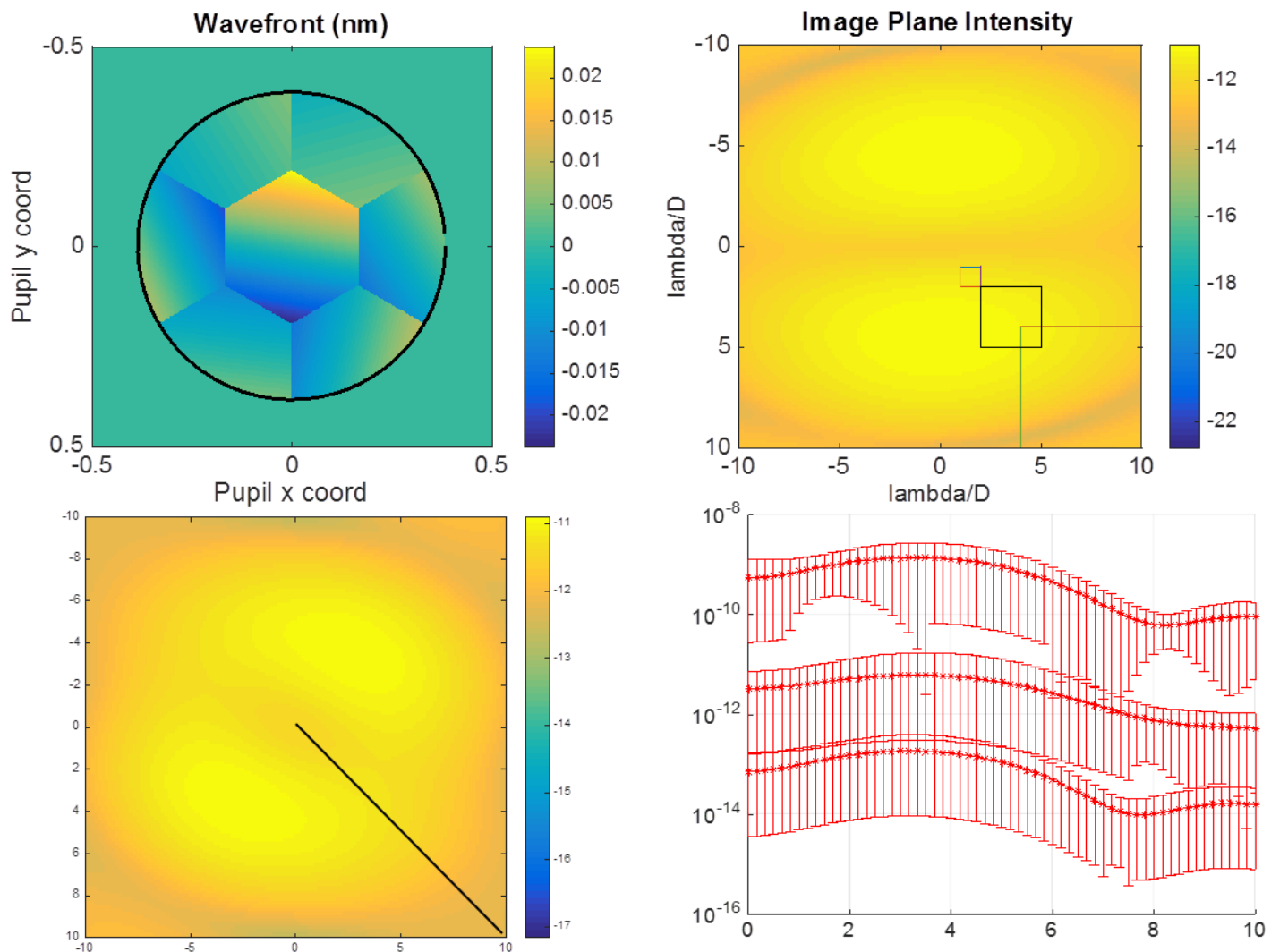
1 Ring Hex Pattern with a Circular Aperture which makes the outer Segment Height half the diameter of the inner Segment.





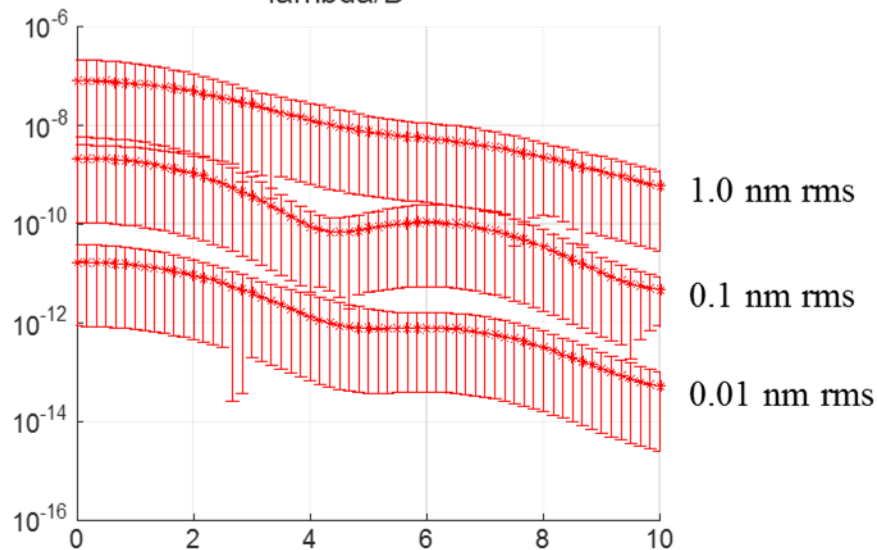
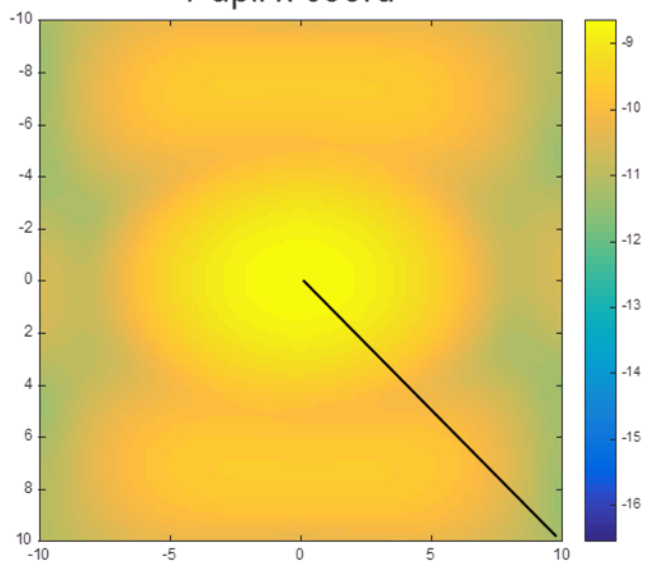
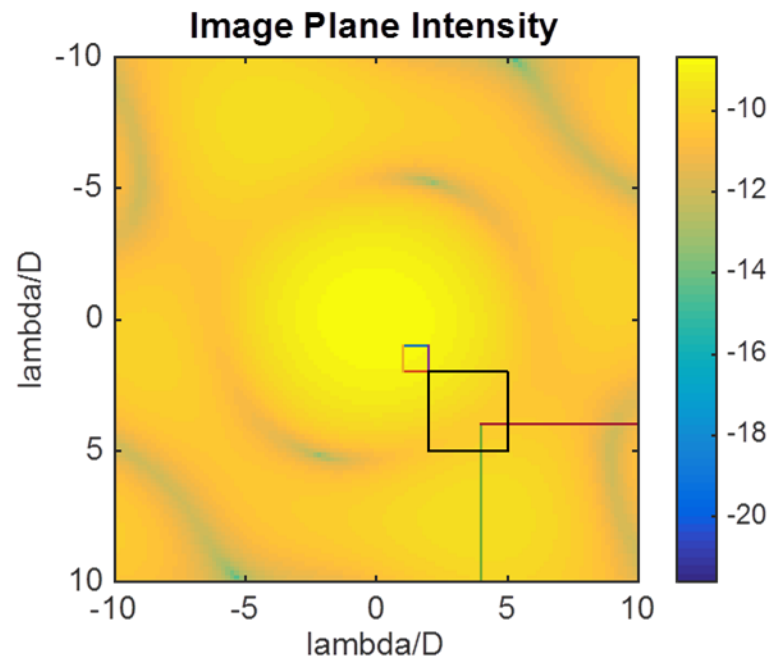
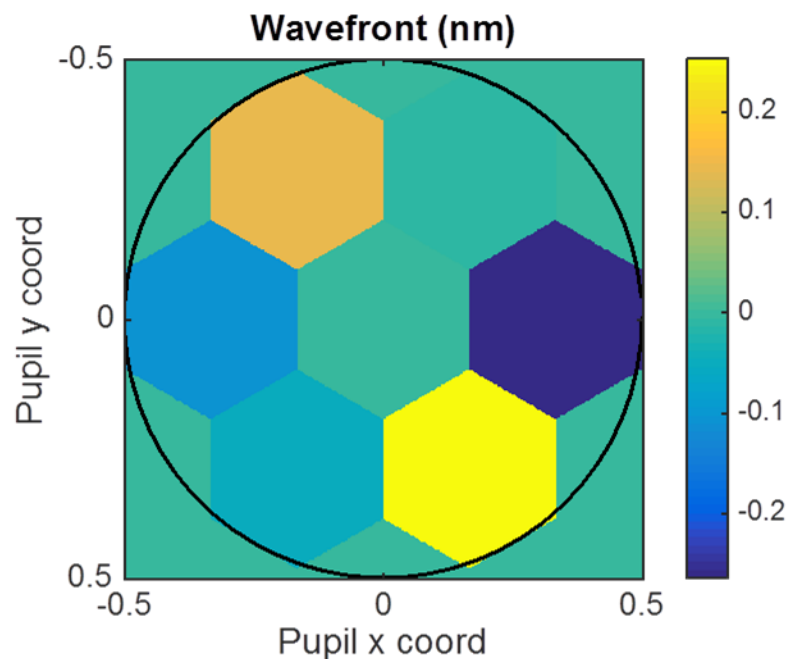
$N_{\text{hex}} = 1 \ 1/0.1/0.01$  rms tilt  $N = 16, \gamma = 6$

1 Ring Hex Pattern with a Circular Aperture which makes the outer Segment Height half the diameter of the inner Segment.





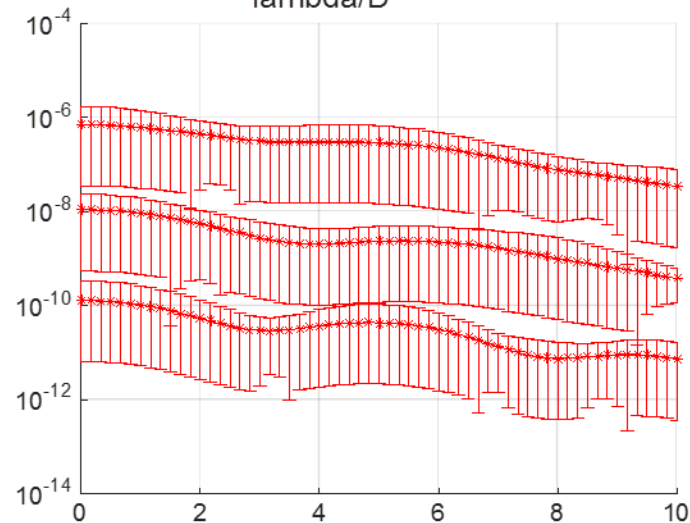
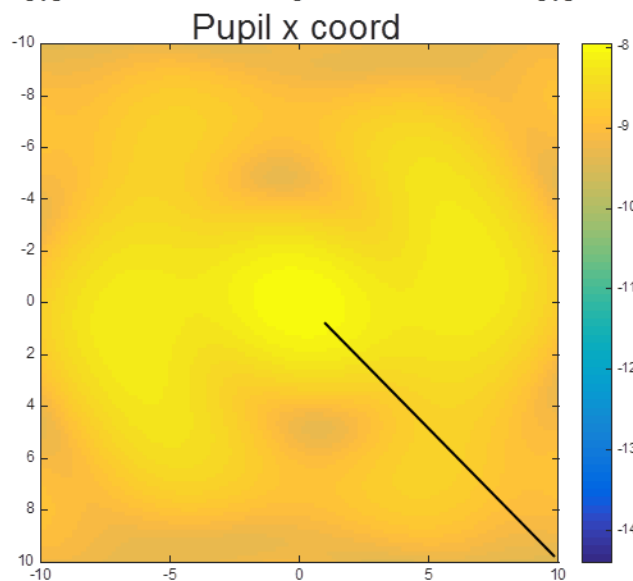
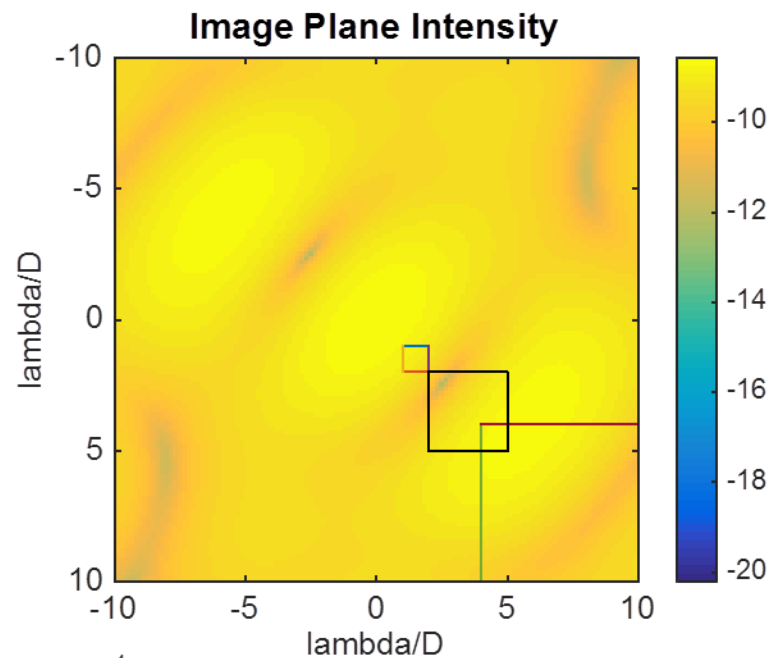
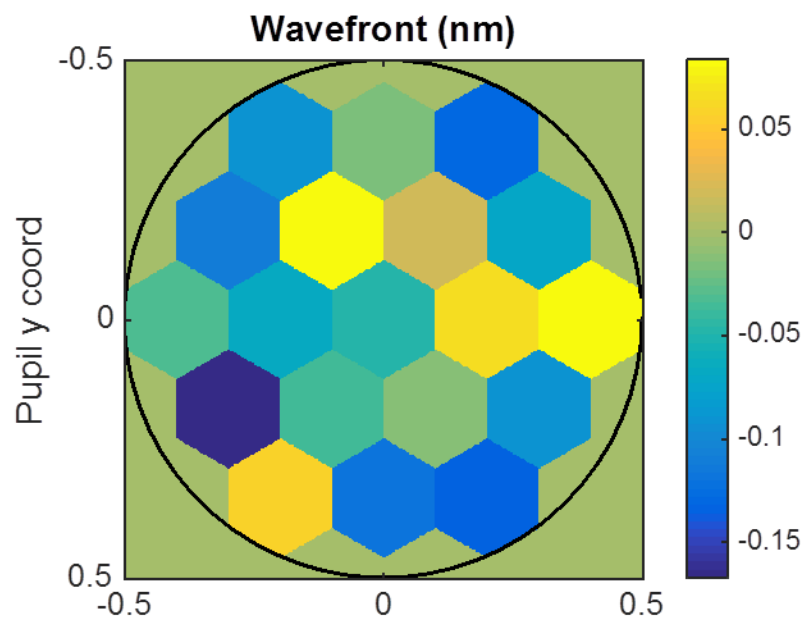
$N_{\text{hex}} = 1 \text{ } 1/0.1/0.01 \text{ rms piston } N = 16, \gamma = 6$





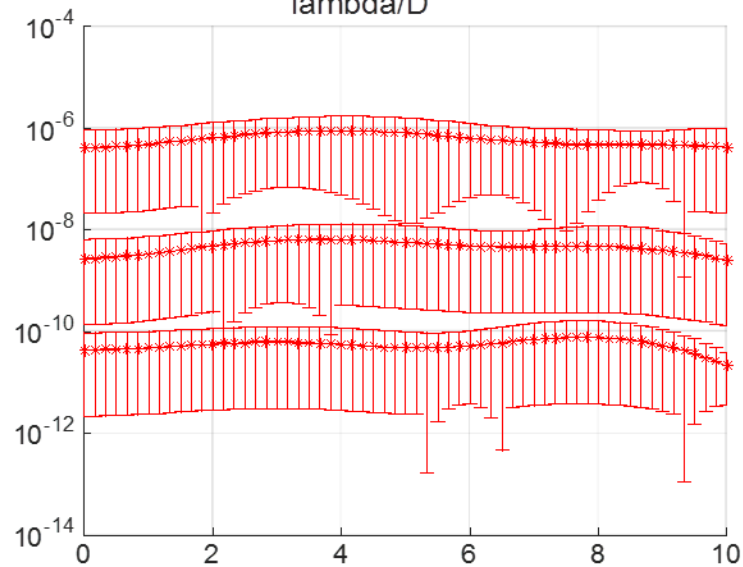
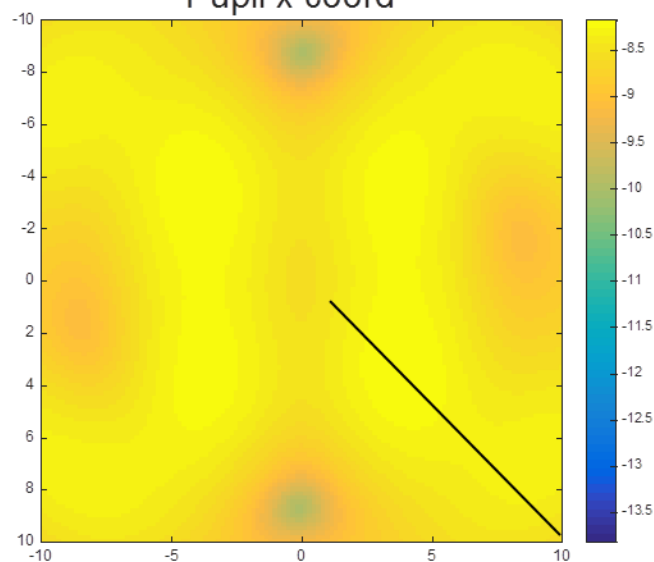
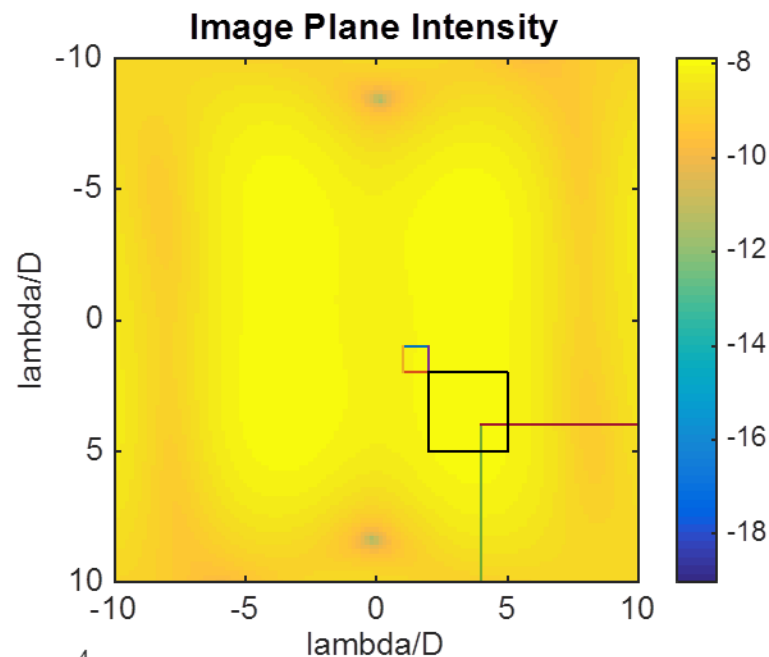
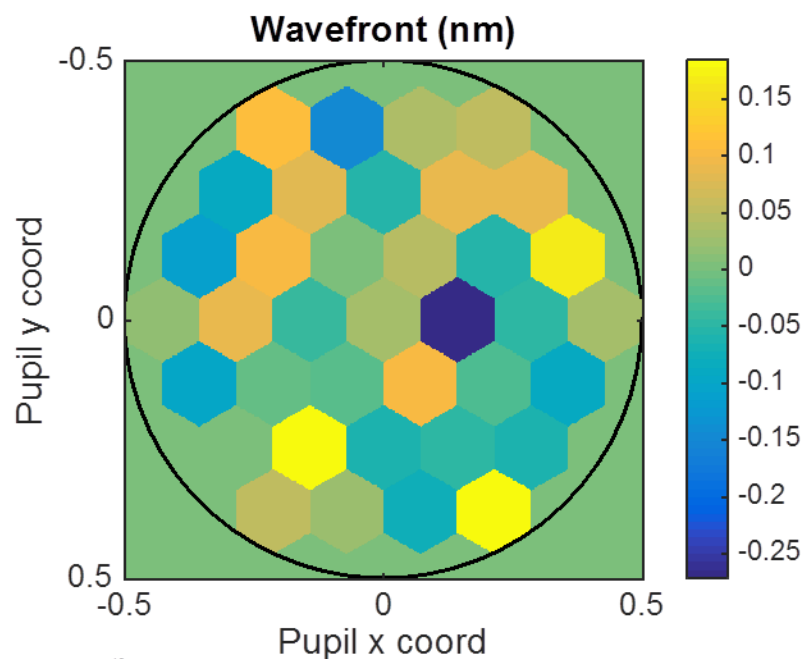


$N_{\text{hex}} = 2 \text{ } 1/0.1/0.01 \text{ rms piston } N = 16, \gamma = 6$



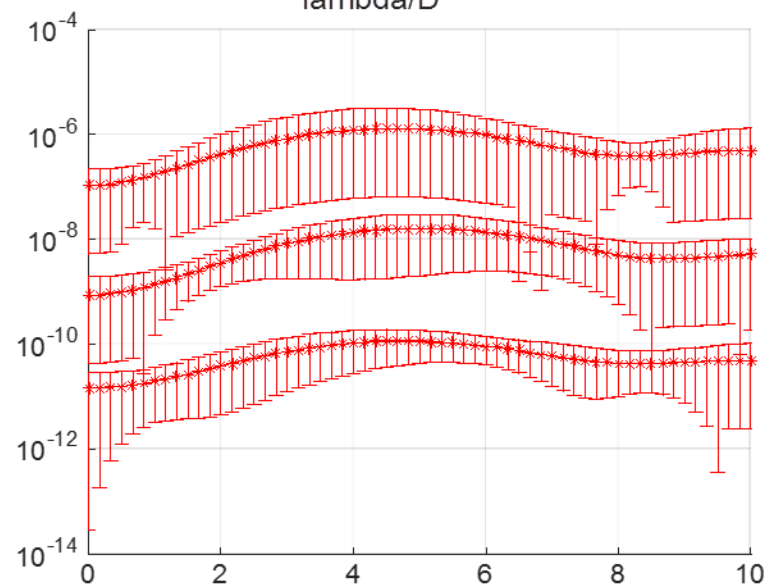
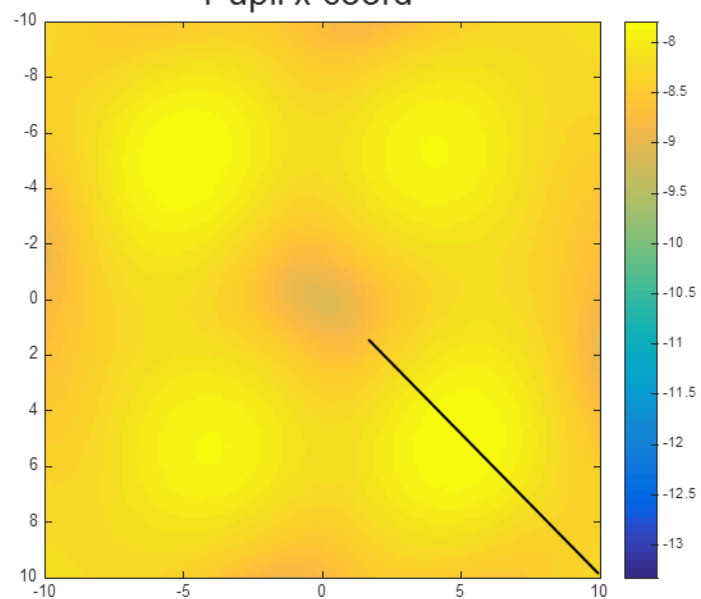
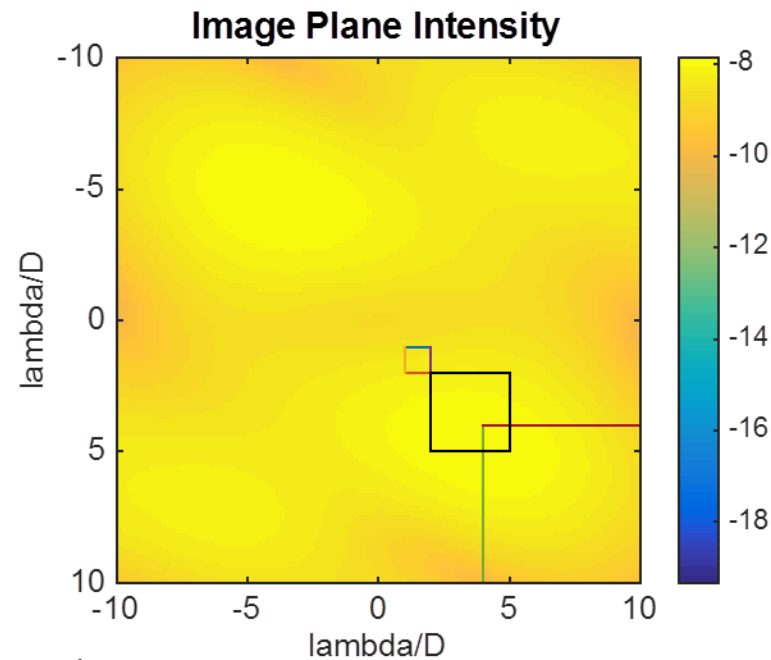
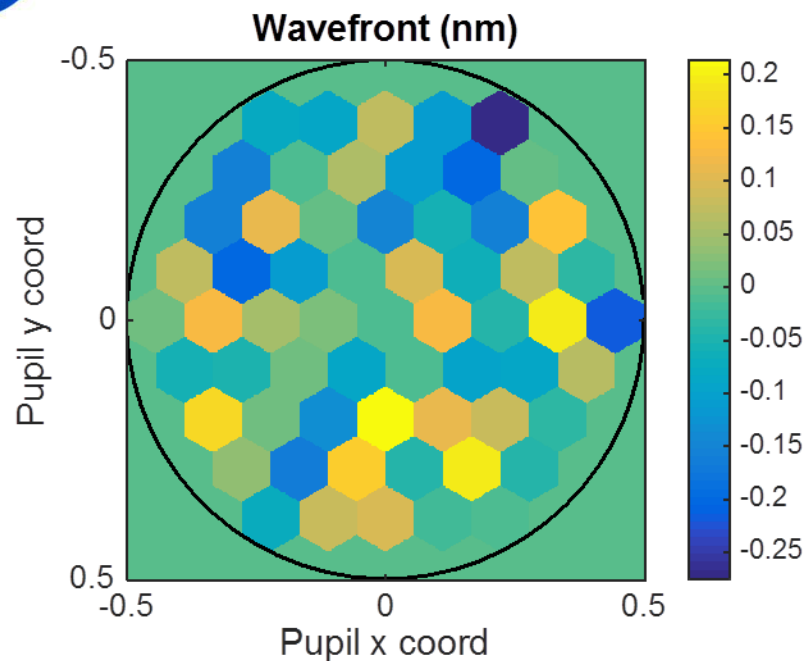


$N_{\text{hex}} = 3 \text{ } 1/0.1/0.01 \text{ rms piston } N = 16, \gamma = 6$



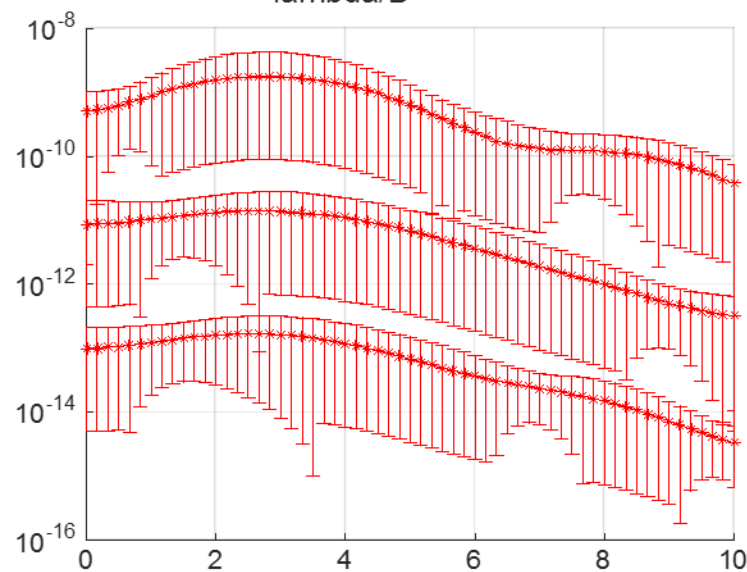
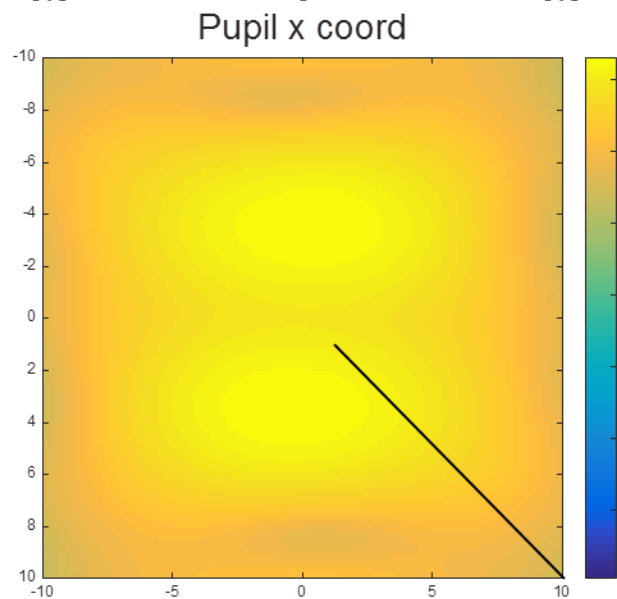
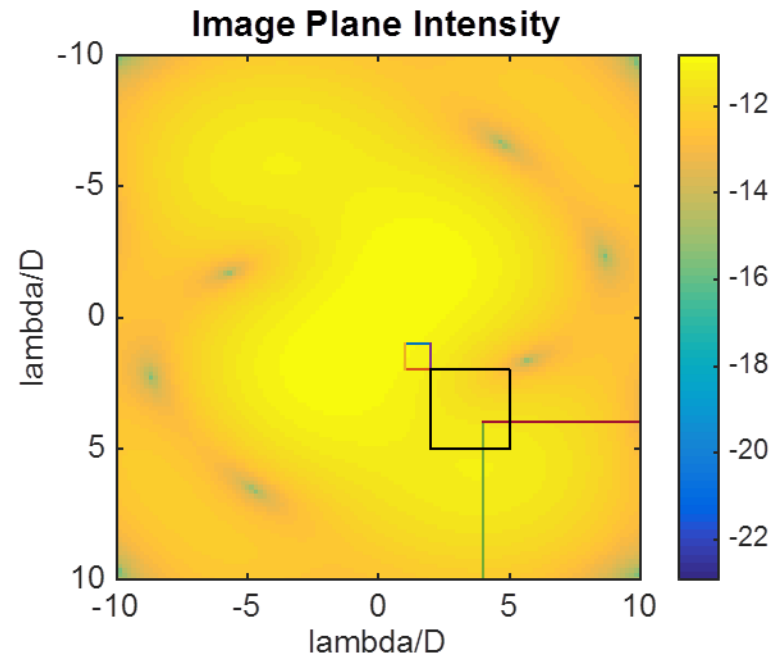
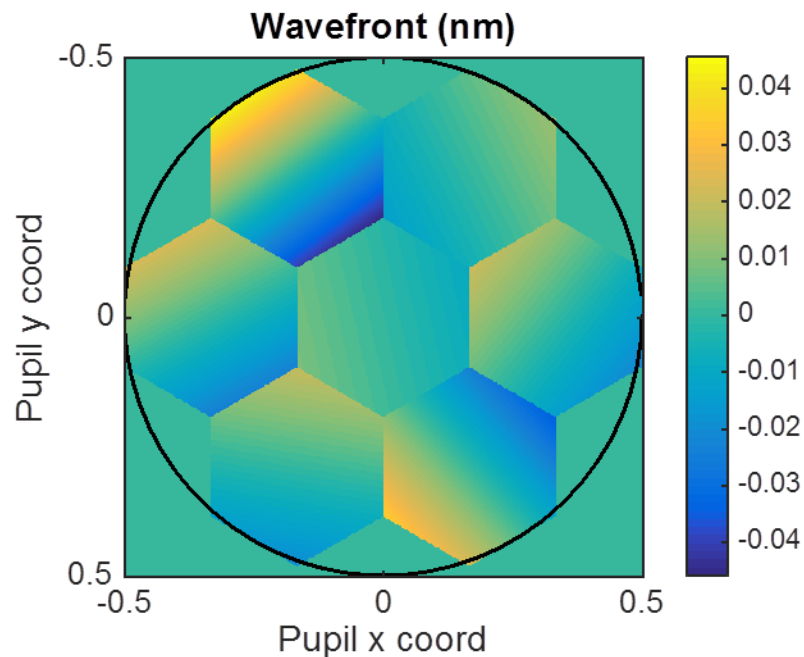


$N_{\text{hex}} = 4 \text{ } 1/0.1/0.01 \text{ rms piston } N = 16, \gamma = 6$



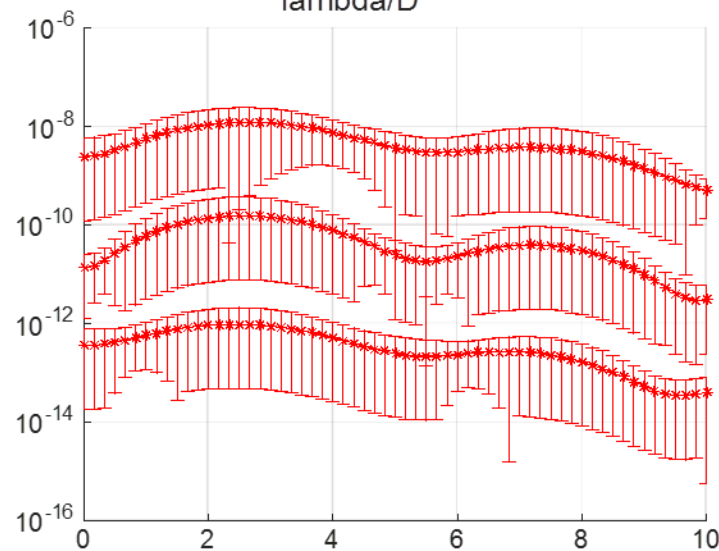
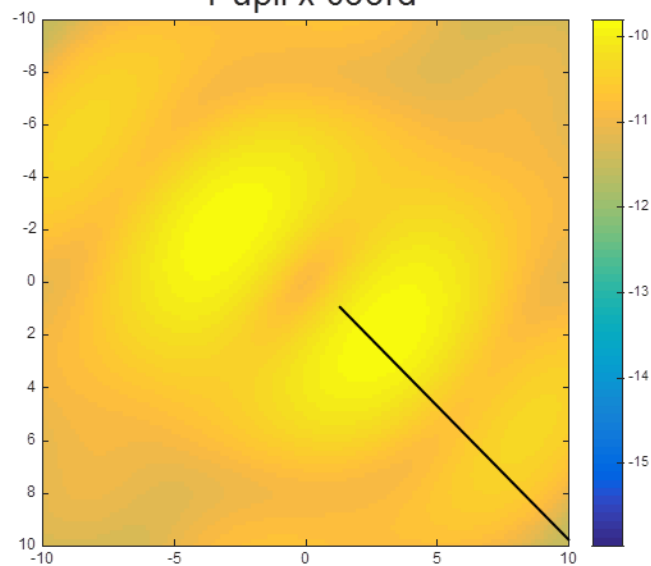
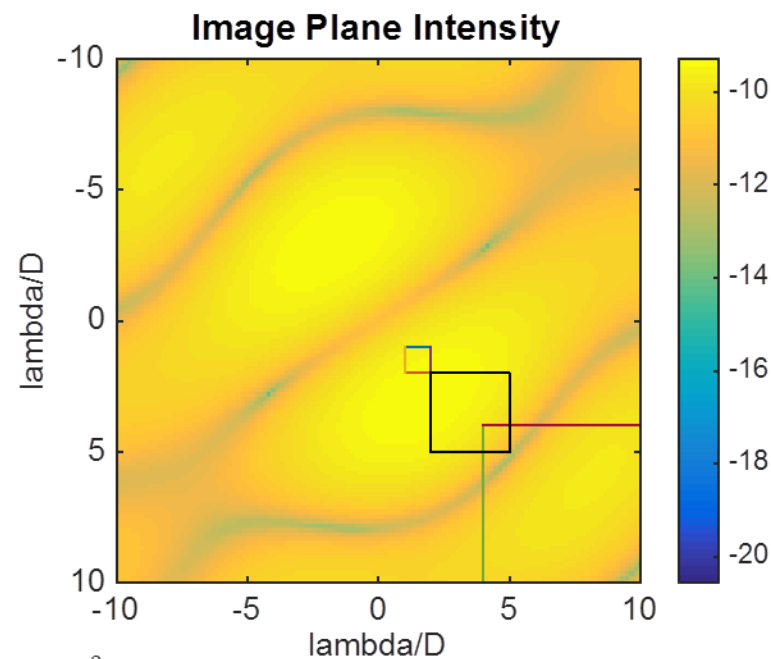
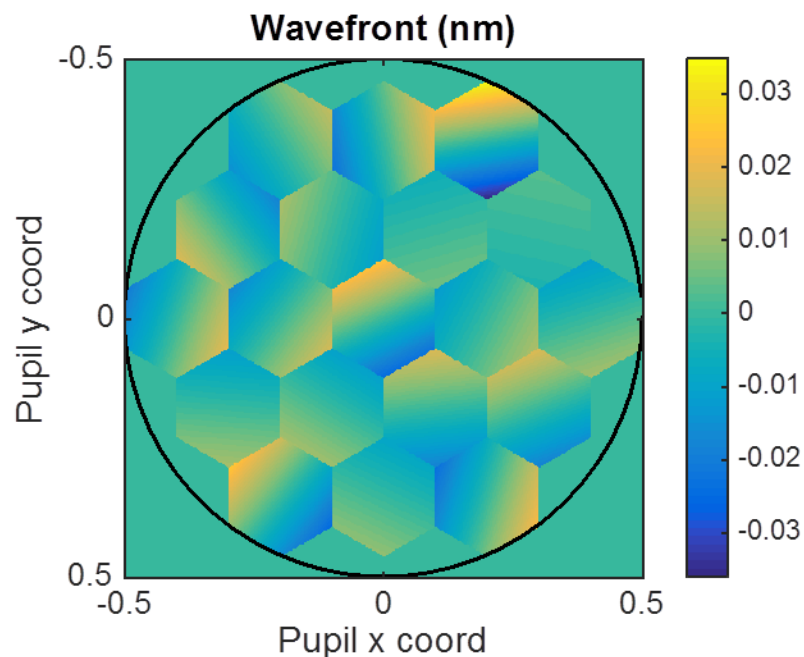


$N_{\text{hex}} = 1 \ 1/0.1/0.01$  rms tip/tilt  $N = 16, \gamma = 6$



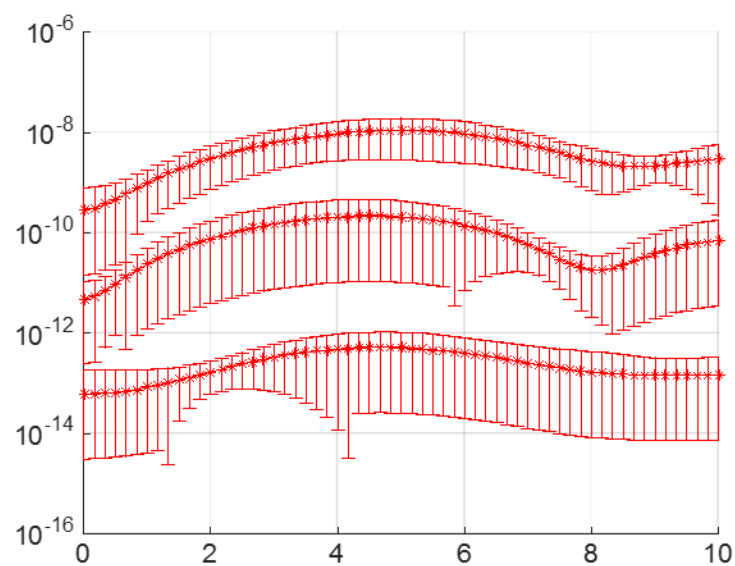
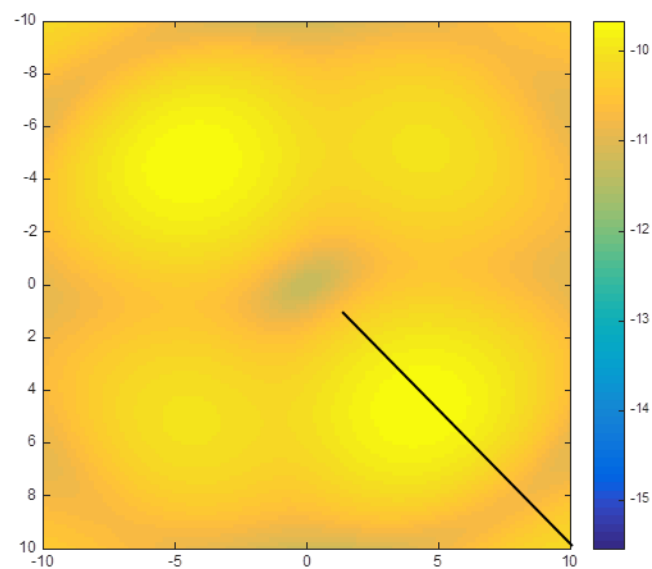
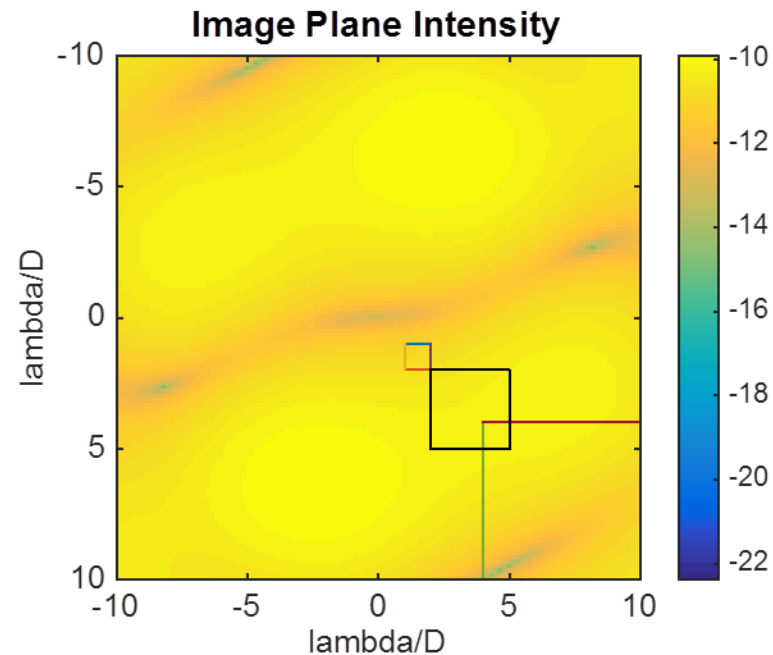
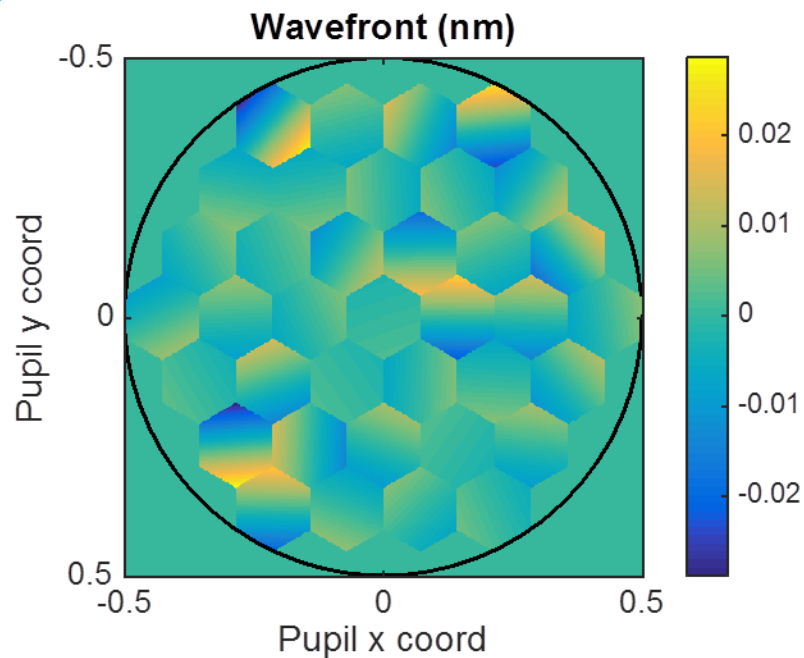


$N_{\text{hex}} = 2 \text{ } 1/0.1/0.01 \text{ rms tip/tilt } N = 16, \gamma = 6$



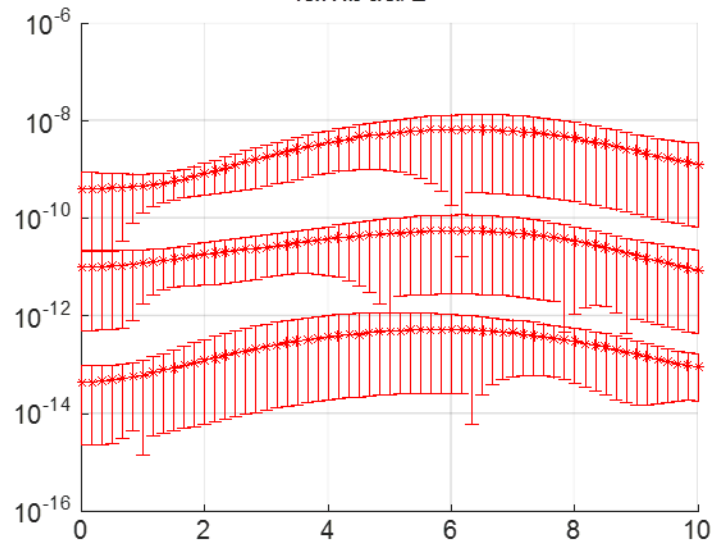
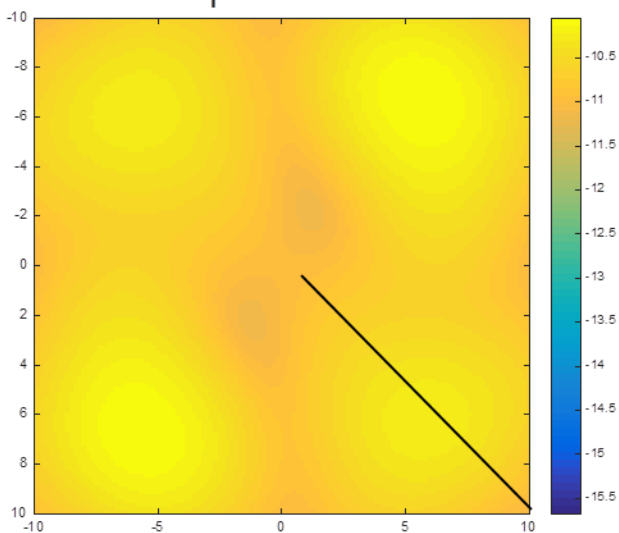
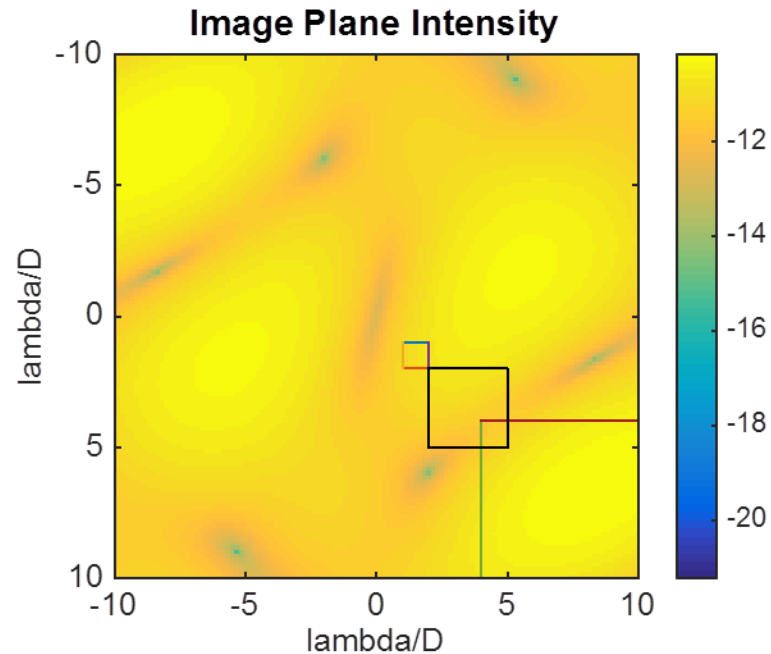
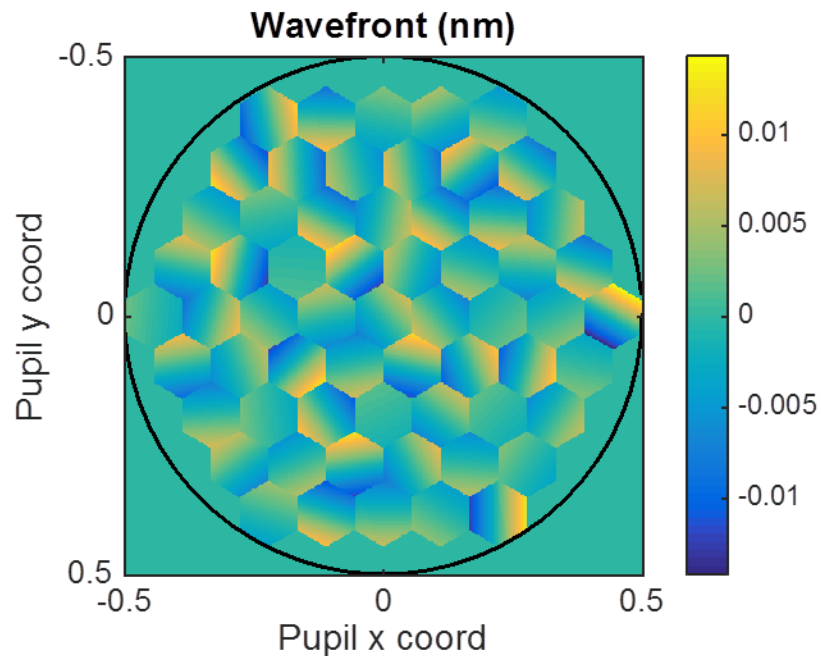


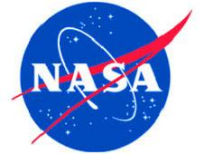
$N_{\text{hex}} = 3 \text{ } 1/0.1/0.01 \text{ rms tip/tilt } N = 16, \gamma = 6$





$N_{\text{hex}} = 4 \text{ } 1/0.1/0.01 \text{ rms tip/tilt } N = 16, \gamma = 6$





## NOTE

Aperture size is selected such that the hex segments have an whole number of pixels in them. This is accomplished via:

$$\text{ap\_size} = (4 * N\_hex + 2) * \text{round}(512 / (4 * N\_hex + 2));$$

Thus the ap sizes for [ 1 2 3 4 5 6 ] are [ 510 510 518 504 506 520 ].